	New Jersey Center for Teaching and Learning	Slide 1 / 168
	Progressive Mathematics Initiative	
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	7th Grade Math		Slide 2 / 168
TEACHING	Ratios & Proportions	TEACHING	
	2013-12-04		
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Writing Ratios	
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	Slide 5 / 168
Ratios	
What do you know about ratios?	
When have you seen or used ratios?	

Ratios	Slide 6 / 168
Ratio - A comparison of two numbers by division	
Ratios can be written three different ways:	
a to b a : b <u>a</u> b	
Each is read, "the ratio of a to b." Each ratio should be in simplest form.	
Find the ratio of boys to girls in this class	

There are 48 animals in the field. Twenty are cows and the rest are horses.	Slide 7 / 168
Write the ratio in three ways:	
a. The number of cows to the number of horses	
b. The number of horses to the number of animals in the field	
Remember to write your ratios in simplest form!	

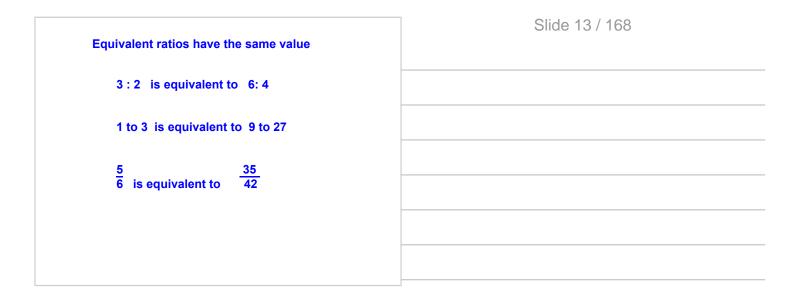
¹ There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of vanilla cupcakes to strawberry cupcakes?	Slide 8 / 168
$ \bigcirc A \ 7:9 \bigcirc B \ \frac{7}{27} \bigcirc C \ \frac{7}{11} \bigcirc D \ 1:3 $	

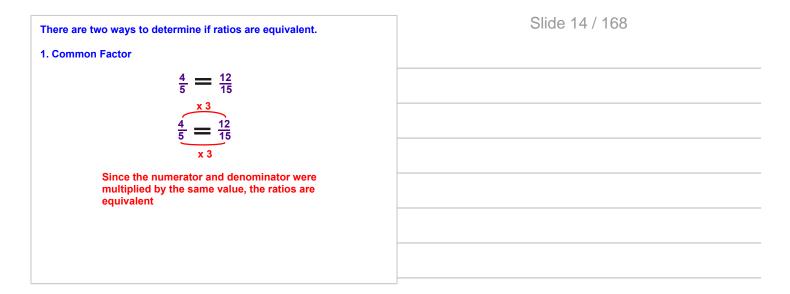
² There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate & strawberry cupcakes to vanilla & chocolate cupcakes?	Slide 9 / 168
$\bigcirc A = \frac{20}{16}$	
$\bigcirc B \frac{11}{7}$ $\bigcirc C \frac{5}{4}$ $\bigcirc D 16$	
○ D <u>16</u> 20	

³ There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of chocolate cupcakes to total cupcakes?	Slide 10 / 168
$ \begin{array}{c} \bigcirc A & \frac{7}{9} \\ \bigcirc B & \frac{7}{27} \\ \bigcirc C & \frac{9}{27} \\ \bigcirc D & \frac{1}{3} \end{array} $	

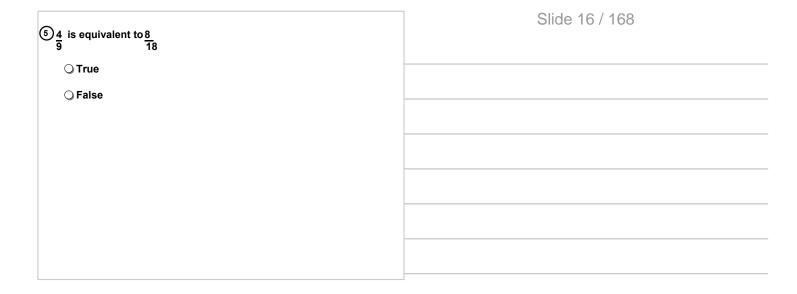
4 There are 27 cupcakes. Nine are chocolate, 7 are vanilla and the rest are strawberry. What is the ratio of total cupcakes to vanilla cupcakes?	Slide 11 / 168
A 27 to 9	
B 7 to 27	
◯ C 27 to 7	
◯ D 11 to 27	

		Slide 12 / 168
Equivalent Ratios		
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	Contents	









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◯ True	
⊖ False	

(7) 18:12 is equivalent to $\frac{9}{6}$, which is equivalent to $\frac{36}{24}$	Slide 18 / 168
◯ True	
⊖ False	

(8) $\frac{2}{24}$ is equivalent to $\frac{10}{120}$, which is equivalent to $\frac{40}{480}$	Slide 19 / 168
◯ True	
⊖ False	

(9) 1:7 is equivalent to $\frac{10}{70}$, which is equivalent to 5 to 65	Slide 20 / 168
◯ True	
⊖ False	

	Slide 21 / 168
Rates	
Return to Table of Contents	

Rates	Slide 22 / 168
Rate: a ratio of two quantities measured in different units	
Examples of rates:	
4 participants/2 teams	
5 gallons/3 rooms	
8 burgers/2 tomatoes	

Unit Rates

Unit rate: Rate with a denominator of one Often expressed with the word "per"

Examples of unit rates:

34 miles/gallon

2 cookies per person

62 words/minute

Slide 23 / 168

Finding a Unit Rate Six friends have pizza together. The bill is \$63. What is the cost per person? Hint: Since the question asks for cost per person, the cost should be first, or in the numerator.

Since unit rates always have a denominator of one, rewrite the rate so that the denominator is one.

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Click for practice. Unit Price Game Are you getting Value For Money? The <u>"Unit Price</u> " tells you the cost per liter, per kilogram, per pound, etc. of what you want to buy. It is a good way of comparing costs. To help you become expert at calculating Unit Prices we	Slide 25 / 168
Nove this game for you:	

¹⁰ Sixty cupcakes are at a party for twenty children. How many cupcakes per person?

	Slide 2	26 / 168	

¹¹ John's car can travel 94.5 miles on 3 gallons of gas. How many miles per gallon can the car travel?	Slide 27 / 168

12	The snake can slither 240 feet in half a day. How many feet	
	can the snake move in an hour?	

¹³ There are five chaperones at the dance of 100 students. How many students per chaperone are there?	Slide 29 / 168

¹⁴ The recipe calls for 6 cups of flour for every four eggs. How many cups of flour are needed for one egg?	Slide 30 / 168

¹⁵ Sarah rode her bike nailes in hour. ³ / ₄ What is	Slide 31 / 168
Sarah's unit rate in miles per hour?	

Slide 32 / 168

We	often use unit rates to easi	ily compare rates.
Exa	mple:	
Seb \$18 earr	astian and Alexandra both astian worked 26 hours on 8.50 before taxes. Alexand ned \$128.25 before taxes. 1	e week and earned Ira worked 19 hours and
at th	heir job? Sebastian	Alexandra

m traveled 480 miles c olds 15 gallons.	on a full tank of gas. His gas tank	Slide 33 /
ara traveled 540 miles nk holds 18 gallons.	on a full tank of gas. Her gas	
hich person's car gets	s better gas mileage?	
Jim	Tara	

¹⁶ Tahira and Brendan going running at the track. Tahira runs 3.5 miles in 28 minutes and Brendan runs 4 miles in 36 minutes. Who runs at a faster pace (miles per hour)?	Slide 34 / 168
Show your work!	
○ A Tahira○ B Brendan	

17	Red apples cost \$3.40 for ten. Green apples cost \$2.46 for six.	Slide 35 / 168
	Which type of apple is cheaper per apple?	
	Show your work!	
	-	
	QA Red apples	
	○ B Green apples	

	Slide 36 / 168
¹⁸ Fruity Oats is \$2.40 for a 12 oz. box.	
Snappy Rice is \$3.52 for a 16 oz. box.	
Which cereal is cheaper per ounce?	
Show your work!	
····· , ····	
○A Fruity Oats	
○ B Snappy Rice	

¹⁹ Two families drive to their vacation spot. The Jones family drives 432 miles and used 16 gallone of gap. The Alwaraz family drives 219	Slide 37 / 168
gallons of gas. The Alverez family drives 319 miles and uses 11 gallons of gas. Which family got more miles per gallon of gas?	
Show your work!	
◯A Jones Family	
◯ B Alverez Family	

20	Mariella typed 123 words in 3 minutes.
	Enrique typed 155 words in 5 minutes.
	Who typed more words per minute?

Show your work!

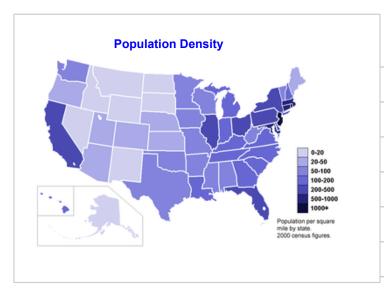
QA Mariella

OB Enrique

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1		
	Population Density	
	Population Density: A unit rate of people per square mile	
	This data is compiled by the US Census Bureau every 10 years and is used when determining the	
	number of Representatives each state gets in the House of Representatives.	

Slide 39 / 16	Sli	de	39	/	1	68
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Click for National Geographic Web Site



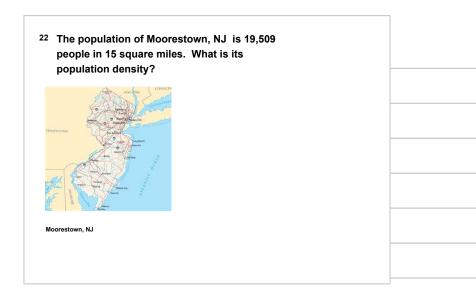
To calculate population density:	Slide 42 / 168
 Find the population of the state. NJ = 8,791,894 people 	
 Find the area of the state. NJ = 7,790 square miles 	
• Divide $\frac{\text{Population}}{\text{Area}} = \frac{8,791,894}{7,790} = 1,129 \text{ people per square mile}$	

We know that New Jersey has 1,129 people per square mile. I compare this data with two oth	Use the links below to	Slide 43 / 168
Population Density	= <u>Population</u> Area	
Click here for population data	Click here for area data Fast Facts Study Guide (State Areas) The table ranke och state by pres (Spare Miles) (petromote hare a <u>Lis declarat Scan</u> e)	
	Rank State Hame Area (Sq JMbs) 1 Alspia 597,70 2 Teas 204,974 3 Colformia 264,974 4 Mostana 147,047 5 Issue Mostana 121,599 6 Artonsia 110,807	
Contraction of the second seco	8 Calerado 104,100 9 Viyoning 67,818 10 Gregon 67,052 11 Usah 64,905 12 Minesedo 84,397 13 Mah 85,574	
	14 Kansas 82,282 15 Nebraska 77,359 16 South Dekota 77,122	

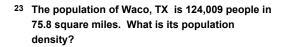
²¹ The population of Newark, NJ is 278,980 people in 24.14 square miles. What is its population density?



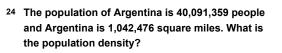
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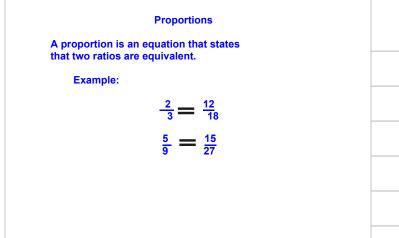
²⁵ The population of San Luis, Argentina is 432,310 people and the Provence is 29,633 square miles. What is the population density?



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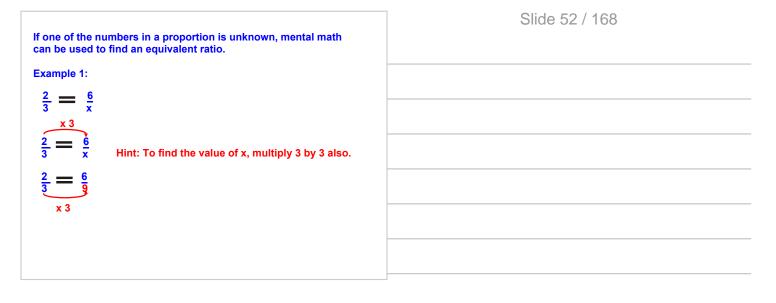
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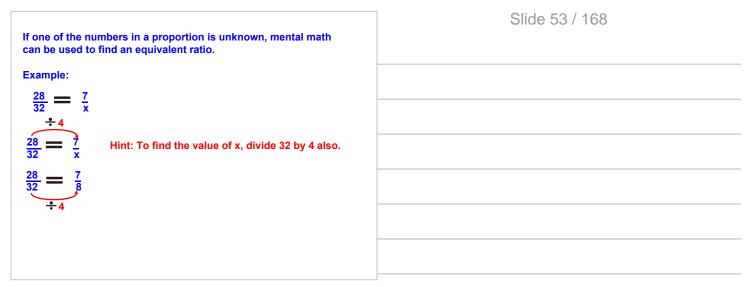




	S	lide 50 /	168	

	Slide 51 / 168
Sorry, this element requires Flash, which is not currently supported in PDFs.	
Please refer to the original Notebook file.	







²⁷ Solve the proportion using equivalent ratios

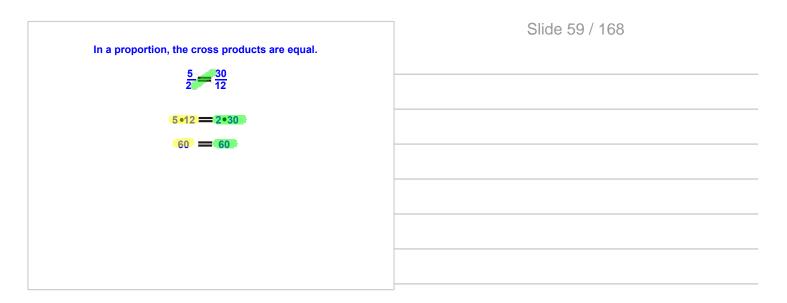
$$\frac{4}{9} = \frac{x}{36}$$

²⁸ Solve the proportion using equivalent ratios	Slide 56 / 168
$\frac{7}{2} = \frac{35}{x}$	

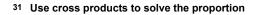
²⁹ Solve the proportion using equivalent ratios	Slide 57 / 168
$\frac{x}{60} = \frac{4}{12}$	

³⁰ Solve the proportion using equivalent ratios





Proportions can also be solved using cross products.		Slide 60 / 168
$\frac{4}{5}$	Cross multiply	
4x = 5 •12		
4x = 60	Solve for x	
x = 15		
Example 2		
$\frac{7}{8}$ $\frac{x}{48}$	Cross multiply	
7•48 = 8x		
336 = 8x	Solve for x	
42 = x		



$$\frac{9}{51} = \frac{x}{17}$$

³² _Use cross products to solve the proportion	Slide 62 / 168
$\frac{x}{12} = \frac{56}{96}$	

33 Use cross products to solve the proportion	Slide 63 / 168
$\frac{45}{18} = \frac{x}{6}$	

34 Use cross products to solve the proportion

$$\frac{2}{15} = \frac{x}{60}$$

Slide 65 / 168

35	Use d	cross	products	to	solve	the	proportion
----	-------	-------	----------	----	-------	-----	------------

$$\frac{7}{x} = \frac{3}{21}$$



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Example.

On a field trip, every chaperone is assigned 12 students. Is the student to chaperone ratio proportional?

If you use a table to demonstrate, you would need several ratios to start.

Chaperones	1	2	3	4	5
Students	12	24	36	48	60

Next, find the simplified ratios and compare them. Are they the same?

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-	

Try this:

The local pizza place sells a plain pie for \$10. Each topping costs an additional \$1.50. Is the cost of pizza proportional to the number of toppings purchased?

Toppings	1	2	3	4
Cost (\$)	11.50	13.00	14.50	16.00

_	 	 	
-			

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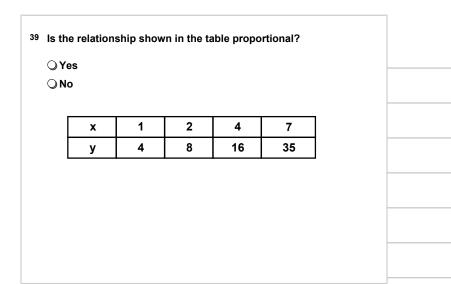
QY	 Is the relationship shown in the table proportional? Yes No 								
	Year Income	1 \$22,000	2 \$44,000	4 \$88,000	5 \$110,000				
	Income	\$22,000	\$44,000	\$88,000	\$110,000				

the relatio	nship shov	wn in the ta	ible propo	ortional?			Slide 71 / 168	
⊖ Yes								
) No								
x	2	5	6	9				
У	7	17.5	21	34.5				

)No
x 1 2 6 9
y 5 11 31 46

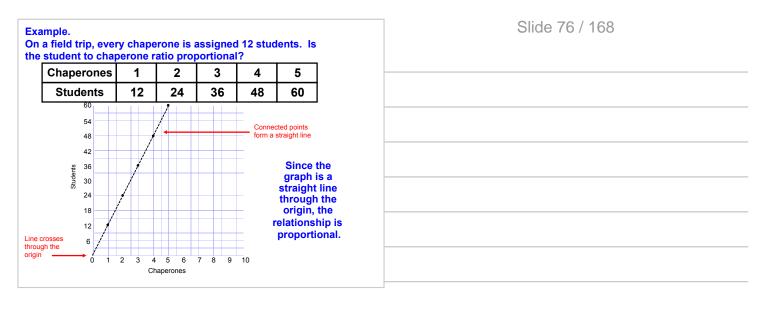
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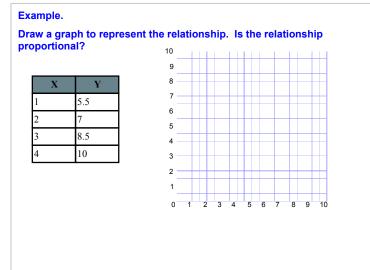
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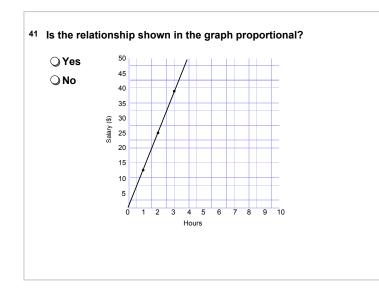
⁴⁰ Is th	e relations	ship show	vn in the ta	able propo	rtional?	 Slide 74 / 168
⊖ Ye ⊖ Ne						
	x y	2 -3	4 -10	6 -15	8 -20	

Remember:	Slide 75 / 168
Table If all the ratios of numbers in the table are equivalent, the relationship is proportional.	
<u>Graph</u> If the graph of the numbers forms a straight line through the origin (0,0), the relationship is proportional.	

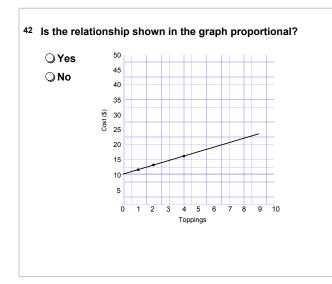




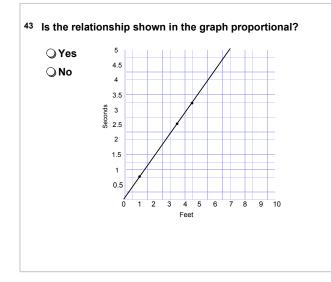




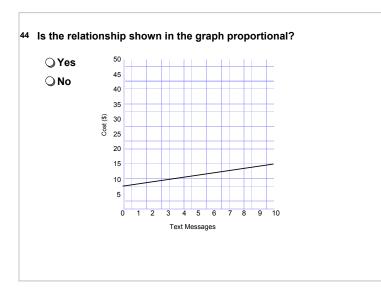




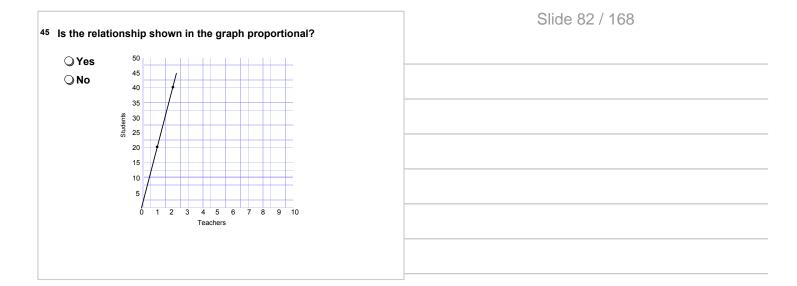














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The constant of proportionality is a constant ratio (unit rate) in any proportional relationship.	
We use the letterk to represent the constant of proportionality.	
Equations:	
$y = kx$ or $k = \frac{y}{x}$	

We can find the constant of proportionality from a table of values, equation and a graph.

In a table, simplify any one of the ratios.

Chaperones	1	2	3	4	5
Students	12	24	36	48	60
		к_ У_	36 _ 1	า	_

$$k = \frac{y}{x} = \frac{30}{3} = 12$$

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the constant	of prop	ortionali	ty:			Slide 86 / 168
Apples (lbs)	2	2.5	3	3.5	4	
Cost (\$)	3.96	4.95	5.94	6.93	7.92	
Click	k					

Find the constant of proportionality:	Slide 87 / 168
X Y 3 4.5 4 6 5 7.5 8 12 9 13.5	
Click	

⁴⁶ Find t	he constant of	f proportionality.	Slide 88 / 168
	X Y]	
2	1.5		
5	3.75	1	
10	7.5]	
12	9		
		-	

⁴⁷ Find the constant of proportionality.	Slide 89 / 168
XY	
2 2.5	
3 3.75	
4 5	
9 11.25	

40		
40	Find the c	onstant o
	X	Y
	50	3
	75	4.5
	100	6
	140	8.4

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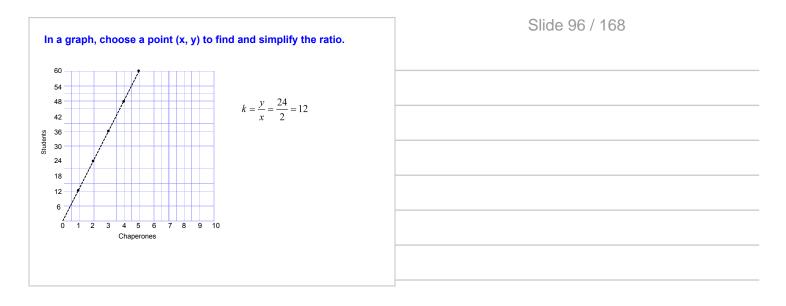


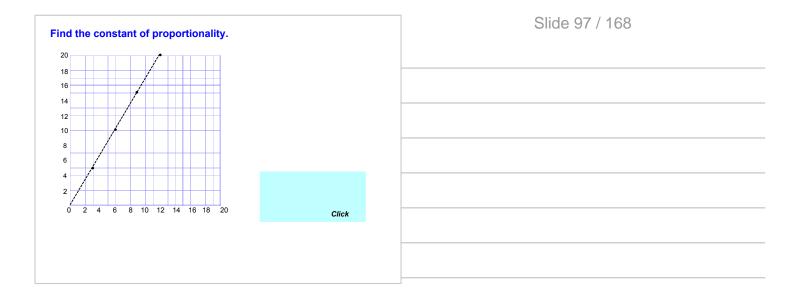
⁵⁰ Find the constant of proportionality.

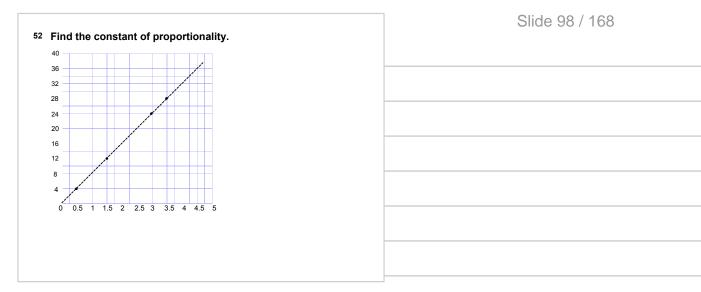
y = 12.9x

51 Find the constant of proportionality. y = 0.45x















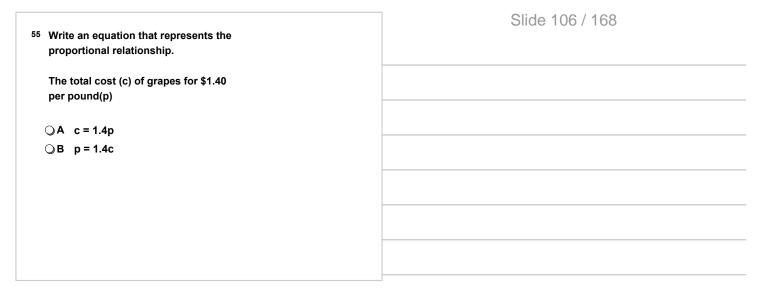
The constant of proportionality and the unit rate are equivalent.	
We can use the constant of proportionality to help write equations using proportional relationships.	
By transforming the equation from: $k = \frac{y}{x}$ to y =/x, we can	
write an equation that can be applied to various situations.	
*Remember: x is the independent variable and y is the dependent variable. This means that a change in x will effect y.	

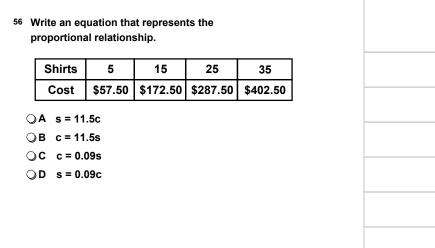
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EXAMPLE	Slide 103 / 168
You are buying Jersey Tomatoes for a cost of 2 pounds for \$3.98. Write an equation to represent the proportional relationship.	
 Let c = cost p = pounds 	
• Determine the unit rate: $k = \frac{3.98}{2} = \frac{1.99}{1}$	
2 1 k = \$1.99 per pound	
 Write an equation to relate the two quantities: c = kp 	
c = 1.99p	

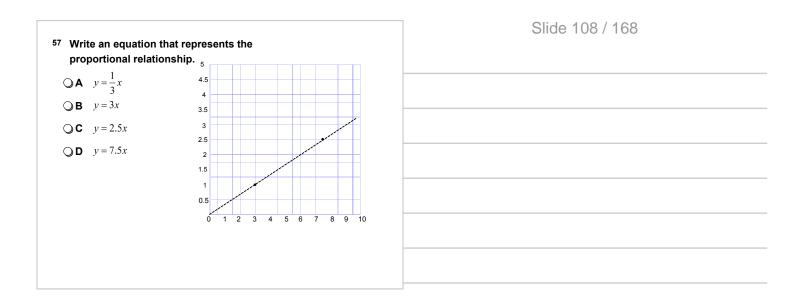


RY THIS:				
Vrite an equation the table.	n to repres	sent the pro	oportional	I relations
Gallons	10	15	20	25
Miles	247	370.5	494	617.5
$k = \frac{m}{g} = \frac{2}{1}$ et g = gallons m = miles	0 = 1			

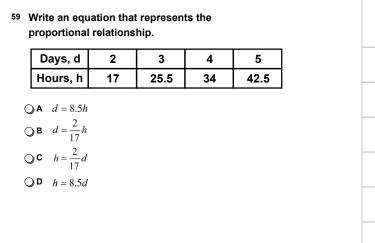








⁵⁸ Write an equation that represents the proportional relationship.	Slide 109 / 168
You are ordering new menus for your restaurant. You pay \$362.50 for 50 menus.	
○ A c = 0.14m	
○ B m = 7.25c	
○ C m = 0.14c	
○ D c = 7.25m	



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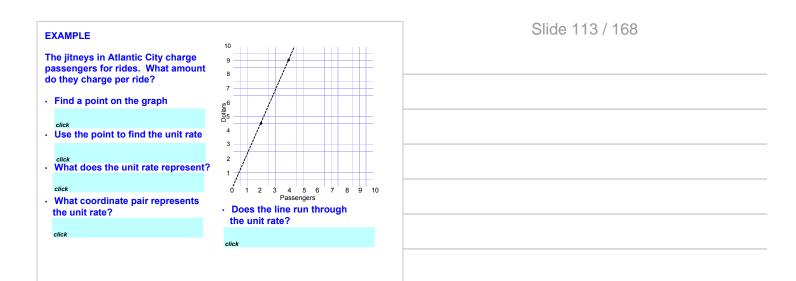


Remember, you can use a graph to determine if a relationship is proportional. How?

If the graph is a straight line going through the origin (0, 0).

Once you determine that the relationship is proportional, you can calculate k, the constant of proportionality. Then, write an equation to represent the relationship.

What do these equations mean? Once we have determined the equation, we can understand what the graph was showing us visually.



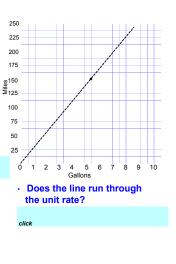
EXAMPLE

Mark drives to work each day. His gas mileage is shown in the graph. What is the unit rate? What does it represent?

- Find a point on the graph
- Use the point to find the unit rate
- · What does the unit rate represent?
- click

click

• What coordinate pair represents the unit rate?



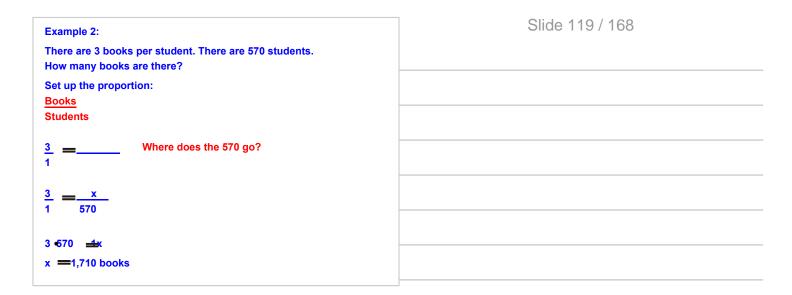








Chocolates at the candy store cost \$6.00 per dozen. How much does one candy cost? Round your answer to the nearest cent.	Slide 118 / 168
Solution:	
$\frac{\$}{\text{candy}} \frac{6.00}{12} = \frac{x}{1} \qquad (Use equivalent rates to set up a proportions)$	
6.00 (1) = 12x	
0.50 = x	
\$0.50 per candy	



Example 3: The ratio of boys How many are gir	to girls is 4 to 5. There are 135 people on a team. ls?	Slide 120 / 168
Set up the propor Girls People	tion: How did we determine this ratio?	
<u>5</u> =	Where does the 135 go?	
$\frac{5}{9} = \frac{x}{-135}$		
5•135 = 9x		
675 = 9x		
x = 75		
75 girls		

⁶⁰ Cereal costs \$3.99 for a one pound box. What is the price per ounce? Round your answer to the nearest penny.	Slide 121 / 168

Bra	ich is the better buy? nd A: \$2.19 for 12 ounces nd B: \$2.49 for 16 ounces
○ A	Brand A
⊖В	Brand B

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62 There are 4 girls for every 10 boys at the party. There are 56 girls at the party. How many boys are there?

63	The farmer has cows and chickens. He owns 5 chickens for
	every cow. He has a total of 96 animals. How many cows
	does he own?

⁶⁴ The auditorium can hold 1 person for every 5 square feet. It is 1210 square feet. How many people can the auditorium hold?	Slide 125 / 168

65	The recipe for one serving calls for 4 oz of beef and 2 oz of bread crumbs. 50 people will be attending the dinner. How many ounces of bread crumbs should be purchased?	Slide 126 / 168

⁶⁶ Mary received 4 votes for every vote that Jane received. 1250 people voted. How many votes did Jane receive?	Slide 127 / 168

⁶⁷ To make the desired shade of pink paint, Brandy uses 3 oz. of red paint for each oz. of white paint. She needs one quart of pink paint. How many oz. of red paint will she need? (1 quart = 32 ounces) Slide 128 / 168

Making Sense of Your Answers	Slide 129 / 168
Sometimes your answer will be a decimal or fraction that may not make sense as an answer. Double check: - Reread the problem - Does your answer make sense? - Do you need to round your answer? - If so, which way should you round your answer?	

68	Cole earned a total of \$11 by selling 8 cups of	
	lemonade. How many cups of lemonade does	
	Cole need to sell in all to earn \$15? Assume the	
	relationship is directly proportional.	

70

69	Hayley learned a total of 13 appetizer recipes over the course of 3 weeks of culinary school. How many weeks does she need to complete to have learned 21 appetizers? Assume the relationship is directly proportional.	Slide 131 / 168

	Slide 132 / 168
Kailyn took a total of 2 quizzes over the course of 5 days. After attending 16 days of school this	
quarter, how many quizzes will Kailyn have taken in total? Assume the relationship is directly proportional.	

71	Brittany baked 18 cookies with 1 cup of flour.
	How many cups of flour does Brittany need in
	order to bake 27 cookies? Assume the
	relationship is directly proportional.

⁷² Shane caught a total of 10 fish over the course of 2 days on a family fishing trip. At the end of what day will Shane have caught his 22 fish? Assume the relationship is directly proportional. Slide 134 / 168

⁷³ In a sample of 50 randomly selected students at a school, 38 students eat breakfast every morning. There are 652 students in the school. Using these results, predict the number of students that eat breakfast.

QA 76

- OB 123
- OC 247
- OD 496

Question from ADP Algebra I End-of-Course Practice Test

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Scale Drawings	
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Scale drawings a	re used to represent objects that are
either too large o	r too small for a life size drawing to
be useful.	

Examples:

A life size drawing of an ant or an atom would be too small to be useful.

A life size drawing of the state of New Jersey or the Solar System would be too large to be useful.

A scale is always provided with a scale drawing.
The <i>scale</i> is the ratio: <u>drawing</u> real life (actual)
When solving a problem involving scale drawings you should:
Write the scale as a ratio
Write the second ratio by putting the provided
information in the correct location (drawing on top & real life on the bottom)
Solve the proportion
and the second

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			 	-
				-
				-

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The distance between Philadelphia and San Francisco
is 2,950 miles. You look on a map and see the scale is
1 inch : 100 miles. What is the distance between the two cities on the map?

drawing <u>1</u> actual 100

Write the scale as a ratio

 $\frac{1}{100} = \frac{x}{2950}$

100x = 2950 x = 29.5

29.5 inches on the map

Try This:	
On a map, the distance between your town and Washington DC is 3.6 inches. The scale is 1 inch : 55 miles. What is the distance between the two cities?	

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On a map with a scale of 1 inch =100 miles, the distance between two cities is 7.55 inches. If a car travels 55 miles per hour, about how long will it take to get from one city to the other.	Slide 142 / 168
 A 13 hrs 45 min. B 14 hrs 30 min. C 12 hrs D 12 hrs 45 min. 	

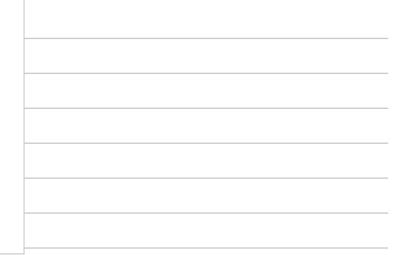
- ⁷⁵ On a map, the scale is 1/2 inch= 300 miles. Find the actual distance between two stores that are 5 1/2 inches apart on the map.
 - \mathbf{A} 3000 miles
 - **OB** 2,727 miles
 - C 3,300 miles
 - $\bigcirc \mathbf{D}$ 1,650 miles

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- ⁷⁶ The figure is a scale of the east side of a house. In the drawing, the side of each square represents 4 feet. Find the width and height of the door. **A** 4 ft by 9 ft **OB** 4 ft by 12 ft
 - **C** 4 ft by 8 ft **O** D 4 ft by 10 ft





77	The distance between Moorestown, NJ		
	and Duck, NC is 910 miles. What is the		
	distance on a map with a scale of 1 inch to		
	110 miles?		

⁷⁸ The distance between Philadelphia and Las Vegas is 8.5 inches on a map with a scale 1.5 in : 500 miles . What is the distance in miles? Slide 146 / 168

⁷⁹ You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the length of the room on the drawing? Slide 147 / 168

80 You are building a room that is 4.6 m long and 3.3 m wide. The scale on the architect's drawing is 1 cm : 2.5 m. What is the width of the room on the drawing?

81	Find the length of a 72 inch wide wall on a scale	Slide 149 / 168
	drawing with a scale 1 inch : 2 feet.	

⁸² You recently purchased a scale model of a car. The scale is 15 cm : 10 m. What is the length of the model car if the real car is 4 m?	Slide 150 / 168

⁸³ You recently purchased a scale model of a car. The scale is 15 cm : 10 m. The length of the model's steering wheel is 1.25 cm. What is the actual length of	Slide 151 / 168
the steering wheel?	



Two objects are similar if they are the same shape but different sizes.

- In similar objects: corresponding angles are congruent corresponding sides are proportional

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