3.2 Parallel Lines and Transversals

**Essential Question** When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

**Exploration 1** Exploring Parallel Lines

Work with a partner.
Use dynamic geometry software to draw two parallel lines. Draw a third line that intersects both parallel lines. Find the measures of the eight angles that are formed. What can you conclude?

**ATTENDING TO PRECISION**
To be proficient in math, you need to communicate precisely with others.

**Exploration 2** Writing Conjectures

Work with a partner. Use the results of Exploration 1 to write conjectures about the following pairs of angles formed by two parallel lines and a transversal.

a. corresponding angles

b. alternate interior angles

c. alternate exterior angles

d. consecutive interior angles

**Communicate Your Answer**

3. When two parallel lines are cut by a transversal, which of the resulting pairs of angles are congruent?

4. In Exploration 2, \( m \angle 1 = 80 \degree \). Find the other angle measures.
What You Will Learn

- Use properties of parallel lines.
- Prove theorems about parallel lines.
- Solve real-life problems.

Using Properties of Parallel Lines

**Theorems**

**Theorem 3.1  Corresponding Angles Theorem**
If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

**Examples** In the diagram at the left, \( \angle 2 \cong \angle 6 \) and \( \angle 3 \cong \angle 7 \).

**Proof** Ex. 36, p. 180

**Theorem 3.2  Alternate Interior Angles Theorem**
If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**Examples** In the diagram at the left, \( \angle 3 \cong \angle 6 \) and \( \angle 4 \cong \angle 5 \).

**Proof** Example 4, p. 134

**Theorem 3.3  Alternate Exterior Angles Theorem**
If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

**Examples** In the diagram at the left, \( \angle 1 \cong \angle 8 \) and \( \angle 2 \cong \angle 7 \).

**Proof** Ex. 15, p. 136

**Theorem 3.4  Consecutive Interior Angles Theorem**
If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

**Examples** In the diagram at the left, \( \angle 3 \) and \( \angle 5 \) are supplementary, and \( \angle 4 \) and \( \angle 6 \) are supplementary.

**Proof** Ex. 16, p. 136

**EXAMPLE 1** Identifying Angles

The measures of three of the numbered angles are 120°. Identify the angles. Explain your reasoning.

**SOLUTION**

By the Alternate Exterior Angles Theorem, \( m\angle 8 = 120^\circ \).

\( \angle 5 \) and \( \angle 8 \) are vertical angles. Using the Vertical Angles Congruence Theorem (Theorem 2.6), \( m\angle 5 = 120^\circ \).

\( \angle 5 \) and \( \angle 4 \) are alternate interior angles. By the Alternate Interior Angles Theorem, \( \angle 4 = 120^\circ \).

So, the three angles that each have a measure of 120° are \( \angle 4, \angle 5, \) and \( \angle 8 \).
EXAMPLE 2 Using Properties of Parallel Lines

Find the value of $x$.

\[ m\angle 4 + (x + 5)° = 180° \]

**SOLUTION**

By the Vertical Angles Congruence Theorem (Theorem 2.6), $m\angle 4 = 115°$. Lines $a$ and $b$ are parallel, so you can use the theorems about parallel lines.

\[ 115° + (x + 5)° = 180° \]

Substitute 115° for $m\angle 4$.

\[ x + 120 = 180 \]

Combine like terms.

\[ x = 60 \]

Subtract 120 from each side.

\[ \text{So, the value of } x \text{ is 60.} \]

EXAMPLE 3 Using Properties of Parallel Lines

Find the value of $x$.

\[ m\angle 1 = (7x + 9)° \]

**SOLUTION**

By the Linear Pair Postulate (Postulate 2.8), $m\angle 1 = 180° - 136° = 44°$. Lines $c$ and $d$ are parallel, so you can use the theorems about parallel lines.

\[ 44° = (7x + 9)° \]

Substitute 44° for $m\angle 1$.

\[ 35 = 7x \]

Subtract 9 from each side.

\[ 5 = x \]

Divide each side by 7.

\[ \text{So, the value of } x \text{ is 5.} \]

Monitoring Progress

Use the diagram.

1. Given $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which theorem you use in each case.

2. Given $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of $x$? Show your steps.
Proving Theorems about Parallel Lines

**EXAMPLE 4** Proving the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

**SOLUTION**

Draw a diagram. Label a pair of alternate interior angles as \( \angle 1 \) and \( \angle 2 \). You are looking for an angle that is related to both \( \angle 1 \) and \( \angle 2 \). Notice that one angle is a vertical angle with \( \angle 2 \) and a corresponding angle with \( \angle 1 \). Label it \( \angle 3 \).

**Given** \( p \parallel q \)

**Prove** \( \angle 1 \cong \angle 2 \)

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( p \parallel q )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 3 )</td>
<td>2. Corresponding Angles Theorem</td>
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<tr>
<td>3. ( \angle 3 \cong \angle 2 )</td>
<td>3. Vertical Angles Congruence Theorem (Theorem 2.6)</td>
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<tr>
<td>4. ( \angle 1 \cong \angle 2 )</td>
<td>4. Transitive Property of Congruence (Theorem 2.2)</td>
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**Monitoring Progress**

3. In the proof in Example 4, if you use the third statement before the second statement, could you still prove the theorem? Explain.

**Solving Real-Life Problems**

**EXAMPLE 5** Solving a Real-life Problem

When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, \( m\angle 2 = 40^\circ \). What is \( m\angle 1 \)? How do you know?

**SOLUTION**

Because the Sun’s rays are parallel, \( \angle 1 \) and \( \angle 2 \) are alternate interior angles. By the Alternate Interior Angles Theorem, \( \angle 1 \cong \angle 2 \).

\[ \angle 1 \cong \angle 2 \]

So, by the definition of congruent angles, \( m\angle 1 = m\angle 2 = 40^\circ \).

**Monitoring Progress**

4. **WHAT IF?** In Example 5, yellow light leaves a drop at an angle of \( m\angle 2 = 41^\circ \). What is \( m\angle 1 \)? How do you know?
1. **WRITING** How are the Alternate Interior Angles Theorem (Theorem 3.2) and the Alternate Exterior Angles Theorem (Theorem 3.3) alike? How are they different?

2. **WHICH ONE DOESN'T BELONG?** Which pair of angle measures does not belong with the other three? Explain.

   - $m\angle 1$ and $m\angle 3$
   - $m\angle 2$ and $m\angle 4$
   - $m\angle 2$ and $m\angle 3$
   - $m\angle 1$ and $m\angle 5$

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**Monitoring Progress and Modeling with Mathematics**

In Exercises 3–6, find $m\angle 1$ and $m\angle 2$. Tell which theorem you use in each case. (See Example 1.)

3. \[117^°\]
   \[\angle 1\] \[\angle 2\]

4. \[150^°\]
   \[\angle 1\] \[\angle 2\]

5. \[122^°\]
   \[\angle 1\] \[\angle 2\]

6. \[140^°\]
   \[\angle 1\] \[\angle 2\]

In Exercises 7–10, find the value of $x$. Show your steps. (See Examples 2 and 3.)

7. \[128^°\]
   \[2x^°\]

8. \[72^°\]
   \[(7x + 24)^°\]

9. \[65^°\]
   \[5(11x - 17)^°\]

10. \[118^°\]
    \[(8x + 6)^°\]

In Exercises 11 and 12, find $m\angle 1$, $m\angle 2$, and $m\angle 3$. Explain your reasoning.

11. \[180^°\]
    \[\angle 1\] \[\angle 2\] \[\angle 3\]

12. \[133^°\]
    \[\angle 1\] \[\angle 2\] \[\angle 3\]

13. **ERROR ANALYSIS** Describe and correct the error in the student’s reasoning.

   \[\angle 9 \cong \angle 10\] by the Corresponding Angles Theorem (Theorem 3.1).
14. **HOW DO YOU SEE IT?**
   Use the diagram.

   a. Name two pairs of congruent angles when \( \overline{AD} \) and \( \overline{BC} \) are parallel. Explain your reasoning.

   b. Name two pairs of supplementary angles when \( \overline{AB} \) and \( \overline{DC} \) are parallel. Explain your reasoning.

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**PROVING A THEOREM** In Exercises 15 and 16, prove the theorem. *(See Example 4.)*

15. Alternate Exterior Angles Theorem (Thm. 3.3)

16. Consecutive Interior Angles Theorem (Thm. 3.4)

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**PROBLEM SOLVING**
A group of campers tie up their food between two parallel trees, as shown. The rope is pulled taut, forming a straight line. Find \( \angle 2 \). Explain your reasoning. *(See Example 5.)*

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18. **DRAWING CONCLUSIONS** You are designing a box like the one shown.

   a. The measure of \( \angle 1 \) is 70°. Find \( \angle 2 \) and \( \angle 3 \).

   b. Explain why \( \angle ABC \) is a straight angle.

   c. If \( \angle 1 \) is 60°, will \( \angle ABC \) still be a straight angle? Will the opening of the box be more steep or less steep? Explain.

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**CRITICAL THINKING** Is it possible for consecutive interior angles to be congruent? Explain.

**THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, is it possible that a transversal intersects two parallel lines? Explain your reasoning.

**MATHEMATICAL CONNECTIONS** In Exercises 21 and 22, write and solve a system of linear equations to find the values of \( x \) and \( y \).

21. \[
   \begin{align*}
   2y^\circ & = 5x^\circ \\
   (14x - 10)^\circ & = 2y^\circ
   \end{align*}
   \]

22. \[
   \begin{align*}
   2y^\circ & = 4x^\circ \\
   (2x + 12)^\circ & = (y + 6)^\circ
   \end{align*}
   \]

**MAKING AN ARGUMENT** During a game of pool, your friend claims to be able to make the shot shown in the diagram by hitting the cue ball so that \( \angle 1 = 25^\circ \). Is your friend correct? Explain your reasoning.

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**MAINTAINING MATHEMATICAL PROFICIENCY**

Write the converse of the conditional statement. Decide whether it is true or false. *(Section 2.1)*

25. If two angles are vertical angles, then they are congruent.

26. If you go to the zoo, then you will see a tiger.

27. If two angles form a linear pair, then they are supplementary.

28. If it is warm outside, then we will go to the park.