Chapter 2

Chapter 2 Maintaining Mathematical Proficiency (p. 63)

1. The first term is 3 and the common difference is 6.

$a_n = a_1 + (n - 1)d$

$a_n = 3 + (n - 1)6$

$a_n = 3 + 6n - 6$

$a_n = 6n - 3$

Use the equation to find a_{50}.

$a_{50} = 6 \cdot 50 - 3 = 300 - 3 = 297$

2. The first term is -29 and the common difference is 17.

$a_n = a_1 + (n - 1)d$

$a_n = -29 + (n - 1)17$

$a_n = -29 + 17n - 17$

$a_n = 17n - 46$

Use the equation to find a_{50}.

$a_{50} = 17 \cdot 50 - 46 = 850 - 46 = 804$

3. The first term is 2.8 and the common difference is 0.6.

$a_n = a_1 + (n - 1)d$

$a_n = 2.8 + (n - 1)0.6$

$a_n = 2.8 + 0.6n - 0.6$

$a_n = 0.6n + 2.2$

Use the equation to find a_{50}.

$a_{50} = 0.6 \cdot 50 + 2.2 = 30 + 2.2 = 32.2$

4. The first term is $\frac{1}{3}$ and the common difference is $\frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$.

$a_n = a_1 + (n - 1)d$

$a_n = \frac{1}{3} + (n - 1)\frac{1}{6}$

$a_n = \frac{2}{6} + (n - 1)\frac{1}{6}$

$a_n = \frac{2}{6} + \frac{1}{6}n - \frac{1}{6}$

$a_n = \frac{1}{6}n + \frac{1}{6}$

Use the equation to find a_{50}.

$a_{50} = \frac{1}{6}(50) + \frac{1}{6} = \frac{51}{6} = 8\frac{1}{2}$

5. The first term is 26 and the common difference is -4.

$a_n = a_1 + (n - 1)d$

$a_n = 26 + (n - 1)(-4)$

$a_n = 26 - 4n + 4$

$a_n = -4n + 30$

Use the equation to find a_{50}.

$a_{50} = -4 \cdot 50 + 30 = -200 + 30 = -170$

6. The first term is 8 and the common difference is -6.

$a_n = a_1 + (n - 1)d$

$a_n = 8 + (n - 1)(-6)$

$a_n = 8 - 6n + 6$

$a_n = -6n + 14$

Use the equation to find a_{50}.

$a_{50} = -6 \cdot 50 + 14 = -300 + 14 = -286$

7.
$$\begin{aligned} 2y - 2x &= 10 \\ -2x &= -2y + 10 \\ \frac{-2}{-2}x &= \frac{-2}{-2}y + \frac{10}{-2} \\ x &= y - 5 \end{aligned}$$

8.
$$\begin{aligned} 20y + 5x &= 15 \\ 5x &= -20y + 15 \\ \frac{5}{5}x &= \frac{-20}{5}y + \frac{15}{5} \\ x &= -4y + 3 \end{aligned}$$

9.
$$\begin{aligned} 4y - 5 &= 4x + 7 \\ 4y - 12 &= 4x \\ \frac{4}{4}y - \frac{12}{4} &= \frac{4}{4}x \\ y - 3 &= x \end{aligned}$$

10.
$$\begin{aligned} y &= 8x - x \\ y &= 7x \\ \frac{y}{7} &= \frac{7x}{7} \\ \frac{1}{7}y &= x \end{aligned}$$

11.
$$\begin{aligned} y &= 4x + zx + 6 \\ y &= x(4 + z) + 6 \\ y - 6 &= x(4 + z) \\ \frac{y - 6}{4 + z} &= \frac{x(4 + z)}{4 + z} \\ \frac{y - 6}{4 + z} &= x \end{aligned}$$

12.
$$\begin{aligned} z &= 2x + 6xy \\ z &= x(2 + 6y) \\ \frac{z}{2 + 6y} &= \frac{x(2 + 6y)}{2 + 6y} \\ \frac{z}{2 + 6y} &= x \end{aligned}$$

13. no; The sequence does not have a common difference.

Chapter 2 Mathematical Practices (p. 64)

1. true

2. flawed; There are no squares that are trapezoids. Trapezoids have only one pair of parallel sides, whereas squares have right angles, congruent sides, and parallel opposite sides.

3. flawed; Only some rectangles are squares.

4. flawed; *ABCD* may be a non-square rectangle.

2.1 Explorations (p. 65)

1. a. true; Thursday always follows Wednesday.

b. false; An acute angle can have any measure greater than 0° and less than 90°.

c. false; The month could be September, April, June, or November.

d. true; All even numbers are divisible by 2, and 9 is not a perfect cube. Because both the hypothesis and conclusion are false, the conditional statement is true.

Chapter 2

2. a. true

$$AB = \sqrt{[-4-(-4)]^2 + (5-0)^2}$$
$$= \sqrt{(-4+4)^2 + 5^2}$$
$$= \sqrt{0+25}$$
$$= \sqrt{25} = 5$$

$$BC = |4-(-4)| = |4+4| = |8| = 8$$

$$AC = \sqrt{(-4-4)^2 + (5-0)^2}$$
$$= \sqrt{(-8)^2 + 5^2}$$
$$= \sqrt{64+25}$$
$$= \sqrt{89}$$

Because $5^2 + 8^2 = \sqrt{89}$, then by the Pythagorean Theorem, $\triangle ABC$ is a right triangle.

b. false

$$BD = \sqrt{[0-(-4)]^2 + (5-0)^2}$$
$$= \sqrt{(0+4)^2 + 5^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

$$DC = \sqrt{[0-(-4)]^2 + (5-0)^2}$$
$$= \sqrt{(0+4)^2 + 5^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

$$BC = |4-(-4)| = 8$$

Because all three sides are not congruent, $\triangle BDC$ is not an equilateral triangle.

c. true

$$BD = \sqrt{[0-(-4)]^2 + (5-0)^2}$$
$$= \sqrt{(0+4)^2 + 5^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

$$DC = \sqrt{[0-(-4)]^2 + (5-0)^2}$$
$$= \sqrt{(0+4)^2 + 5^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16+25}$$
$$= \sqrt{41}$$

$$BC = |4-(-4)| = 8$$

Because at least two sides are congruent, $\triangle BDC$ is an isosceles triangle.

d. true

Slope of $\overline{AD}$: $\dfrac{5-5}{0-(-4)} = \dfrac{0}{4} = 0$

Slope of $\overline{BC}$: $\dfrac{0-0}{4-(-4)} = \dfrac{0}{8} = 0$

Because the slope of $\overline{AD}$ is equal to the slope of $\overline{BC}$, $\overline{AD} \parallel \overline{BC}$ and $ABCD$ is a trapezoid.

e. false

Slope of $\overline{AB}$ is undefined.

Slope of $\overline{DC}$: $\dfrac{5-0}{0-4} = \dfrac{5}{-4}$

$\overline{AB}$ is not parallel to $\overline{DC}$, because the slopes are not equal.

3. a. true; The Pythagorean Theorem is valid for all right triangles.

b. false; Two angles are complementary when the sum of their measures is 90°.

c. false; The sum of the angle measures of a quadrilateral is always 360°.

d. true; Collinear points are points on the same line.

e. true; Every pair of intersecting lines forms two pairs of opposite rays and therefore two pairs of vertical angles.

4. A conditional statement is true if both the hypothesis and the conclusion are true or if both are false or if a false hypothesis yields a true conclusion. A conditional statement is false when a true hypothesis yields a conclusion that is false.

5. *Sample answer:*

True: If two angles are supplementary, then the sum of the angles is 180°.

False: If two angles have a sum of 180°, then the angles form a linear pair. (The angles may have a sum of 180° but not be adjacent angles.)

2.1 Monitoring Progress (pp. 66–70)

1. Hypothesis: All 30° angles

Conclusion: Acute angles

If-then form: If an angle measures 30°, then it is an acute angle.

2. Hypothesis: $x = -3$

Conclusion: $2x + 7 = 1$

If-then form: If $x = -3$, then $2x + 7 = 1$.

3. The shirt is not green. **4.** The shoes are red.

5. a. Conditional: If the stars are visible then it is night; true.

b. Converse: If it is night, then the stars are visible; false (could be cloudy).

c. Inverse: If the stars are not visible, then it is not night; false (could be cloudy).

d. Contrapositive: If it is not night, then the stars are not visible; true.

6. true; The diagram shows that $\angle JMF$ and $\angle FMG$ are a linear pair. By definition, angles that form a linear pair are supplementary.

7. false; The midpoint cannot be assumed from a diagram without markings that indicate $FM = MH$.

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8. true; Because M lies on $\overleftrightarrow{FH}$ and $\overleftrightarrow{JG}$, two pairs of opposite rays are formed.

9. false; Right angles and perpendicular lines cannot be assumed from a diagram without being marked as such.

10. An angle is a right angle if and only if its measure is 90°.

11. Two line segments have the same length if and only if they are congruent segments.

12. Mary is in the fall play if and only if she is taking theater class.

13. You can run for President if and only if you are at least 35 years old.

14.

p	q	~q	p→~q
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

15.

p	q	p→q	~(p→q)
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

2.1 Exercises (pp. 71–74)

Vocabulary and Core Concept Check

1. A conditional statement and its contrapositive, as well as the converse and inverse of a conditional statement are both true or both false.

2. The statement that does not belong is "If you are an athlete, then you play soccer." This statement is false and the other three are true.

Monitoring Progress and Modeling with Mathematics

3. Hypothesis: A polygon is a pentagon.
   Conclusion: It has five sides.

4. Hypothesis: Two lines form vertical angles.
   Conclusion: They intersect.

5. Hypothesis: You run.
   Conclusion: You are fast.

6. Hypothesis: You like math.
   Conclusion: You like science.

7. If $x = 2$, then $9x + 5 = 23$.

8. If today is Friday, then tomorrow is the weekend.

9. If you are in a band, then you play the drums.

10. If two angles are right angles, then they are supplementary.

11. If you are registered, then you are allowed to vote.

12. If two angles are complementary, then their measures sum to 90°.

13. The sky is not blue.

14. The lake is not cold.

15. The ball is pink.

16. The dog is a lab.

17. Conditional statement: If two angles are supplementary, then the measures of the angles sum to 180°; true.
    Converse: If the measures of two angles sum to 180°, then the two angles are supplementary; true.
    Inverse: If two angles are not supplementary, then their measures do not sum to 180°; true.
    Contrapositive: If the measures of two angles do not sum to 180°, then they are not supplementary; true.

18. Conditional statement: If you are in a math class, then you are in Geometry; false.
    Converse: If you are in Geometry, then you are in math class; true.
    Inverse: If you are not in math class, then you are not in Geometry; true.
    Contrapositive: If you are not in Geometry, then you are not in math class; false.

19. Conditional statement: If you do your math homework, then you will do well on your test; false.
    Converse: If you do well on your test, then you did your math homework; false.
    Inverse: If you do not do your math homework, then you will not do well on your test; false.
    Contrapositive: If you do not do well on your test, then you did not do your math homework; false.

20. Conditional statement: If you are not an only child, then you have a sibling; true.
    Converse: If you have a sibling, then you are not an only child; true.
    Inverse: If you are an only child, then you do not have a sibling; true.
    Contrapositive: If you do not have a sibling, then you are an only child; true.

21. Conditional statement: If it does not snow, then I will run outside; false.
    Converse: If I run outside, then it is not snowing; true.
    Inverse: If it snows, then I will not run outside; true.
    Contrapositive: If I do not run outside, then it is snowing; false.

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22. Conditional statement: If the Sun is out, then it is daytime; true.

Converse: If it is daytime, then the Sun is out; false.

Inverse: If the Sun is not out, then it is not daytime; false.

Contrapositive: If it is not daytime, then the Sun is not out; true.

23. Conditional statement: If $3x - 7 = 20$, then $x = 9$; true.

Converse: If $x = 9$, then $3x - 7 = 20$; true.

Inverse: If $3x - 7 \neq 20$, then $x \neq 9$; true.

Contrapositive: If $x \neq 9$, then $3x - 7 \neq 20$; true.

24. Conditional statement: If it is Valentine's Day, then it is February; true.

Converse: If it is February, then it is Valentine's Day; false.

Inverse: If it is not Valentine's Day, then it is not February; false.

Contrapositive: If it is not February, then it is not Valentine's Day; true.

25. true; By definition of right angle, the measure of the right angle shown is 90°.

26. true; If two lines form a right angle, then the lines are perpendicular.

27. true; If two adjacent angles form a linear pair, then the sum of the measures of the two angles is 180°.

28. false; The midpoint cannot be assumed unless $\overline{AM}$ and $\overline{MB}$ are marked as congruent.

29. A point is the midpoint of a segment if and only if the point divides the segment into two congruent segments.

30. Two angles are vertical angles if and only if their sides form two pairs of opposite rays.

31. Two angles are adjacent angles if and only if they share a common vertex and side, but have no common interior points.

32. Two angles are supplementary angles if and only if the sum of the two angle measures is 180°.

33. A polygon has three sides if and only if it is a triangle.

34. A polygon is a quadrilateral if and only if it has four sides.

35. An angle is a right angle if and only if the angle measures 90°.

36. An angle has a measure between 90° and 180° if and only if it is obtuse.

37. Taking four English courses is a requirement regardless of the total amount of courses the student takes, and the courses do not have to be taken simultaneously. The correct if-then form is: If students are in high school, then they will take four English courses before they graduate.

38. The inverse was used instead of the converse. The correct converse is: If I bring an umbrella, then it is raining.

39.

p	~p	q	~p→q
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F

40.

p	q	~q	~q→p
T	T	F	T
T	F	T	T
F	T	F	T
F	F	T	F

41.

p	~p	q	~q	~p→~q	~(~p→~q)
T	F	T	F	T	F
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	T	T	F

42.

p	q	~q	p→~q	~(p→~q)
T	T	F	F	T
T	F	T	T	F
F	T	F	T	F
F	F	T	T	F

43.

p	q	~p	q→~p
T	T	F	F
T	F	F	T
F	T	T	T
F	F	T	T

44.

p	q	q→p	~(q→p)
T	T	T	F
T	F	T	F
F	T	F	T
F	F	T	F

45. **a.** If a rock is igneous, then it is formed from the cooling of molten rock.

If a rock is sedimentary, then it is formed from pieces of other rocks.

If a rock is metamorphic, then it is formed by changing temperature, pressure, or chemistry.

b. If a rock is formed from the cooling of molten rock, then it is igneous; true; All rocks formed from cooling molten rock are called igneous.

If a rock is formed from pieces of other rocks, then it is sedimentary; true; All rocks formed from pieces of other rocks are called sedimentary.

If a rock is formed by changing temperature, pressure, or chemistry, then it is metamorphic; true; All rocks formed by changing temperature, pressure, or chemistry are called metamorphic.

c. *Sample answer:* If a rock is not sedimentary, then it was not formed from pieces of other rocks; This is the inverse of one of the conditional statements in part (a). So, the converse of this statement will be the contrapositive of the conditional statement. Because the contrapositive is equivalent to the conditional statement and the conditional statement was true, the contrapositive will also be true.

46. A biconditional statement is true only if the conditional and converse statements are both true. The shirt could have been purchased at another location other than the mall, so the sister is correct.

47. no; The contrapositive is equivalent to the original conditional statement. In order to write a conditional statement as a true biconditional statement, you must know that the converse (or inverse) is true.

48. The if-then statement is the inverse of the conditional statement:

Conditional statement: $p \rightarrow q$; inverse: $\sim p \rightarrow \sim q$

49. If you tell the truth, then you don't have to remember anything.

Hypothesis: You tell the truth.

Conclusion: You don't have to remember anything.

50. If you expect things of yourself, then you can do them.

Hypothesis: You expect things of yourself.

Conclusion: You can do them.

51. If one is lucky, then a solitary fantasy can totally transform one million realities.

Hypothesis: One is lucky.

Conclusion: A solitary fantasy can totally transform one million realities.

52. If you are happy, then you will make others happy too.

Hypothesis: You are happy.

Conclusion: You will make others happy too.

53. no; "If $x^2 - 10 = x + 2$, then $x = 4$" is a false statement because $x = -3$ is also possible. The converse, however, of the original conditional statement is true. In order for a biconditional statement to be true, both the conditional statement and its converse must be true.

54. a. *Sample answer:* If a natural arch is the largest in the United States, then it is the Landscape Arch. If a natural arch is the Landscape Arch, then it spans 290 feet.

b. Contrapositive: If a natural arch is not the Landscape Arch, then it is not the largest in the United States. If a natural arch does not span 290 feet, then it is not the Landscape Arch.

c. Converse: If a natural arch is the Landscape Arch, then it is the largest in the United States.

Inverse: If a natural arch is not the largest in the United States, then it is not the Landscape Arch.

Both of these statements are true because there is only one arch that fits both criteria.

Converse: If a natural arch spans 290 feet, then it is the Landscape Arch.

Inverse: If a natural arch is not the Landscape Arch, then it does not span 290 feet.

Both of these statements are false because it is possible for a natural arch in another country to span 290 feet.

55. A; You can rewrite the given statement in if-then form as: If you do your homework, then you can watch a movie afterward.

56. *Sample answer:*

If $4x = 28$, then $x = 7$. (true)

If $5y = 25$, then $y = 4$. (false)

If 6 times your age is subtracted from 5 times my age, then the result is 0. (Whether the statement is true or false depends on the ages of the people. If your age is 15 and my age is 18, then this statement is true, however if your age is 18 and my age is 15, then this statement is false.)

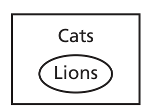
57. If yesterday was February 28, then today is March 1.

58. *Sample answer:* If a person is in chorus, then the person is a musician.

If a person is in jazz band, then the person is in band.

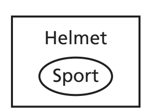
If a person is in band, then the person is a musician.

59. a.



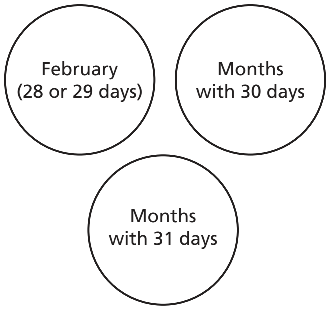
If you see a cat, then you went to the zoo to see a lion; The original statement is true, because a lion is a type of cat, but the converse is false, because you could see a cat without going to the zoo.

b.



If you wear a helmet, then you play a sport; Both the original statement and the converse are false, because not all sports require helmets and sometimes helmets are worn for activities that are not considered a sport, such as construction work.

c.



If this month is not February, then it has 31 days; The original statement is true, because February never has 31 days, but the converse is false, because a month that is not February could have 30 days.

60. **a.** true (as long as $x \neq y$)

b. If the mean of the data is between x and y, then x and y are the least and greatest values in your data set. This converse is false, because x and y could be any two values in the set as long as one is higher and one is lower than the mean.

c. If a data set has a mean, median, and a mode, then the mode of the data set will always be a data value. The mean is always a calculated value that is not necessarily equal to any of the data values, and the median is a calculated value when there are an even number of data values. The mode is the data value with the greatest frequency, so it is always a data value.

61. *Sample answer:*

Conditional statement: If the course is Biology, then the class is a science class.

Converse: If the class is a science class, then the course is Biology.

62. By definition of linear pairs, $\angle 1$ and $\angle 2$ are supplementary. So, if $m\angle 1 = 90°$, then $m\angle 2 = 90°$. Also, by definition of linear pairs, $\angle 2$ and $\angle 3$ are supplementary. So, if $m\angle 2 = 90°$, then $m\angle 3 = 90°$. Finally, by definition of linear pairs, $\angle 3$ and $\angle 4$ are supplementary. So, if $m\angle 3 = 90°$, then $m\angle 4 = 90°$.

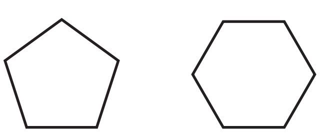
63. *Sample answer:*

Slogan: "This treadmill is a fat-burning machine!"

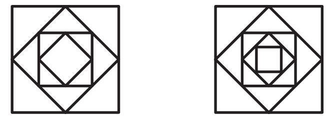
Conditional statement: If you use this treadmill, then you will burn fat quickly.

Maintaining Mathematical Proficiency

64. The pattern is to add a side to the previous polygon.



65. The pattern is to add a square that connects the midpoints of the previously added square.



66. The pattern is to add 2 to the previous number.

1

$1 + 2 = 3$

$3 + 2 = 5$

$5 + 2 = 7$

$7 + 2 = 9$

$9 + 2 = 11$

The next two numbers in the pattern are 9 and 11.

67. The pattern is to add 11 to the previous number:

12

$12 + 11 = 23$

$23 + 11 = 34$

$34 + 11 = 45$

$45 + 11 = 56$

$56 + 11 = 67$

The next two numbers in the pattern are 56 and 67.

68. The pattern is to multiply the previous number by $\frac{2}{3}$:

2

$2 \cdot \frac{2}{3} = \frac{4}{3}$

$\frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9}$

$\frac{8}{9} \cdot \frac{2}{3} = \frac{16}{27}$

$\frac{16}{27} \cdot \frac{2}{3} = \frac{32}{81}$

$\frac{32}{81} \cdot \frac{2}{3} = \frac{64}{243}$

The next two numbers in the pattern are $\frac{32}{81}$ and $\frac{64}{243}$.

69. The pattern is n^2, where $n \geq 1$.

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

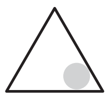
$6^2 = 36$

The next two numbers in the pattern are 25 and 36.

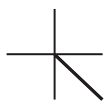
Chapter 2

2.2 Explorations (p. 75)

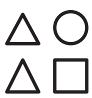
1. a. The circle is rotating from one vertex in the triangle to the next in a clockwise direction.



b. The pattern alternates between a curve in an odd quadrant and a line segment with a negative slope in an even quadrant. The quadrants with a curve or a line segment follow the pattern I, IV, III, II, and the curves follow the pattern of two concave down and two concave up.



c. The pattern alternates between the first three arrangements, then their respective mirror images.



2. a. true; Because all of Property B is inside Property A, all items with Property B must also have Property A.

b. false; There is a region for items that have Property A but not B.

c. false; There is a region for items that have Property A but not C.

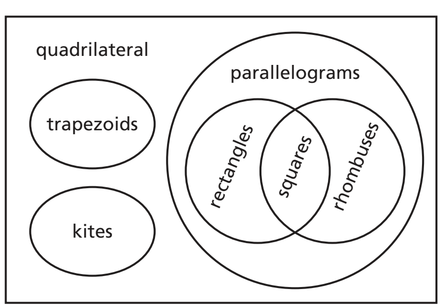
d. true; There is a region for items that have Property A but not B.

e. true; There is no intersection of the regions for Properties C and B.

f. true; There is a region that is the intersection of Properties A and C.

g. false; There is no intersection of the regions for Properties B and C.

3.



Sample answer:

If a quadrilateral is a rectangle, then it is a parallelogram.

If a quadrilateral is a kite, then it is not a parallelogram.

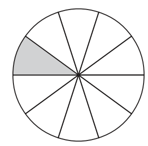
If a quadrilateral is a square, then it is a rhombus, a rectangle, and a parallelogram.

4. You can look for a pattern and then use a "rule" based on that pattern to predict what will happen if the pattern continues.

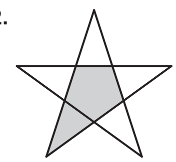
5. *Sample answer:* You noticed that you did much better on your math tests when you were able to study for at least one hour the night before as opposed to when you were only able to study for less than an hour. So now you make sure that you study for at least one hour the night before a test.

2.2 Monitoring Progress (pp. 76–79)

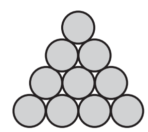
1. Divide the circle into 10 equal parts. Shade the section just above the horizontal segment on the left.



2.



3. Add 4 circles at the bottom.



4. The product of any three negative integers will yield a negative integer.

Tests: $(-2) \cdot (-6) \cdot (-4) = -48$

$(-5) \cdot (-2) \cdot (-1) = -10$

5. The sum of any five consecutive integers is 5 times the middle (third) number.

Tests: $2 + 3 + 4 + 5 + 6 = 20 = 5(4)$

$-2 + (-1) + 0 + 1 + 2 = 0 = 5(0)$

6. *Sample answer:* If $x = \frac{1}{2}$, then $x^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$. $\frac{1}{4}$ is less than $\frac{1}{2}$, not greater.

7. *Sample answer:* The sum of -1 and -3 is $-1 + (-3) = -4$. The difference of -1 and -3 is $-1 - (-3) = -1 + 3 = 2$. Because $-4 < 2$, the sum is not greater than the difference.

8. $\angle R$ is obtuse.

9. If you get an A on your math test, then you can watch your favorite actor.

Chapter 2

10. Conjecture: The sum of a number and itself is 2 times the number. If n is the number, then $n + n = 2n$.

Inductively: $4 + 4 = 8$, $10 + 10 = 20$,
$45 + 45 = 90$, $n + n = 2n$

Deductively: Let n be any number. By the Reflexive Property, $n = n$. If n is added to each side by the Addition Property, then $n + n = n + n$. Combining like terms yields $2n = 2n$. Therefore, $n + n = 2n$, which means the sum of any number and itself is 2 times the number.

11. Deductive reasoning is used because the Law of Detachment is used to reach the conclusion.

2.2 Exercises (pp. 80–82)

Vocabulary and Core Concept Check

1. Because the prefix *counter-* means "opposing," a counterexample opposes the truth of the statement.

2. Inductive reasoning uses patterns to write a conjecture. Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

Monitoring Progress and Modeling with Mathematics

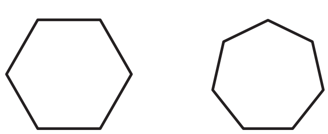
3. The absolute value of each number in the list is 1 greater than the absolute value of the previous number in the list, and the signs alternate from positive to negative. The next two numbers are: -6, 7.

4. The numbers are increasing by successive multiples of 2. The sequence is: $0 + 2 = 2$, $2 + 4 = 6$, $6 + 6 = 12$, $12 + 8 = 20$, $20 + 10 = 30$, $30 + 12 = 42$, etc. So, the next two numbers are: 30, 42.

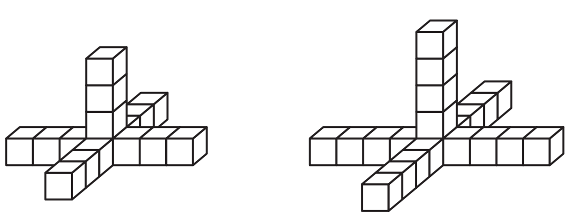
5. The pattern is the alphabet written backward. The next two letters are: U, T.

6. The letters represent the first letter of each month of the year, and they are in the order of the months. The next two letters are: J, J.

7. The pattern is regular polygons having one more side than the previous polygon.



8. The pattern is the addition of 5 blocks to the previous figure. One block is added to each of the four ends of the base and one block is added on top. So, the next two figures will have 16 blocks and then 21 blocks.



9. The product of any two even integers is an even integer.

Tests: $2 \cdot 8 = 16$, $22 \cdot 20 = 440$

10. The sum of an even integer and an odd integer is an odd integer.

Tests: $3 + 4 = 7$, $6 + 13 = 19$

11. The quotient of a number and its reciprocal is the square of that number.

Tests: $\dfrac{10}{\left(\frac{1}{10}\right)} = \dfrac{10}{1} \cdot \dfrac{10}{1} = 100 = 10^2$

$$\frac{\left(\frac{2}{3}\right)}{\left(\frac{3}{2}\right)} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$$

12. The quotient of two negative numbers is a positive rational number.

Tests: $\dfrac{-24}{-12} = 2$, $\dfrac{-33}{-3} = 11$

13. *Sample answer:* Let the two positive numbers be $\frac{1}{2}$ and $\frac{1}{6}$.

The product is $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$. Because $\frac{1}{12} < \frac{1}{2}$ and $\frac{1}{12} < \frac{1}{6}$, the product of two positive numbers is not always greater than either number.

14. *Sample answer:* Let $n = -1$.

$$\frac{-1 + 1}{-1} = 0$$

$$0 \ngtr 1$$

15. Each angle could be 90°. Then neither are acute.

16. If line s intersects $\overline{MN}$ at any point other than the midpoint, it is not a segment bisector.

17. You passed the class.

18. not possible; You may get to the movies by other means.

19. not possible; $QRST$ could be a rectangle.

20. P is the midpoint of $\overline{LH}$.

21. not possible

22. If $\frac{1}{2}a = 1\frac{1}{2}$, then $5a = 15$.

23. If a figure is a rhombus, then the figure has two pairs of opposite sides that are parallel.

24. not possible

25. The law of logic used was the Law of Syllogism.

26. The law of logic used was the Law of Detachment.

27. The law of logic used was the Law of Detachment.

28. The law of logic used was Law of Syllogism.

29. $1 + 3 = 4, \quad 3 + 5 = 8, \quad 7 + 9 = 16$

Conjecture: The sum of two odd integers is an even integer. Let m and n be integers, then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1) + (2n + 1) &= 2m + 2n + 2 \\ &= 2(m + n + 1)\end{aligned}$$

Any number multiplied by 2 is an even number. Therefore, the sum of two odd integers is an even integer.

30. $1 \cdot 3 = 3, \quad 3 \cdot 5 = 15, \quad 7 \cdot 9 = 63$

Conjecture: The product of two odd integers is an odd integer. Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1)(2n + 1) &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1\end{aligned}$$

Any number multiplied by 2 is an even number, and adding 1 will yield an odd number. Therefore, the product of two odd integers is an odd integer.

31. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

32. deductive reasoning; The conclusion is based on mathematical definitions and properties.

33. deductive reasoning; Laws of nature and the Law of Syllogism were used to draw the conclusion.

34. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

35. The Law of Detachment cannot be used because the hypothesis is not true; *Sample answer:* Using the Law of Detachment, because a square is a rectangle, you can conclude that a square has four sides.

36. The conjecture was based on a pattern in specific cases, not rules or laws about the general case; Using inductive reasoning, you can make a conjecture that you will arrive at school before your friend tomorrow.

37. Using inductive reasoning, you can make a conjecture that male tigers weigh more than female tigers because this was true in all of the specific cases listed in the table.

38. a. yes; Bases on inductive reasoning, the pattern in all of the years shown is that the number of girls participating is more than the year before.

b. no; There is no information in the graph about how the number of girl participants compares with the number of boy participants.

39. 1: $2 = 1(2)$

2: $2 + 4 = 6 = 2(3)$

3: $2 + 4 + 6 = 12 = 3(4)$

4: $2 + 4 + 6 + 8 = 20 = 4(5)$

5: $2 + 4 + 6 + 8 + 10 = 30 = 5(6)$

$\vdots \qquad \vdots$

n: $n(n + 1)$

So, the sum of the first n positive even integers is $n(n + 1)$.

40. a. $1 + 1 = 2, 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8,$ $8 + 5 = 13, 13 + 8 = 21, 21 + 13 = 34$

Each number in the sequence is the sum of the previous two numbers in the sequence.

b. $21 + 34 = 55$

$34 + 55 = 89$

$55 + 89 = 144$

c. *Sample answer:* A spiral can be drawn by connecting the opposite corners of squares with side lengths that follow the Fibonacci sequence. This spiral is similar to the spiral seen on nautilus shells. It is also similar to the golden spiral, which is sometimes found in spiraling galaxies.

41. Argument 2: This argument uses the Law of Detachment to say that when the hypothesis is met, the conclusion is true.

42. Pattern 1: Multiply each term by 2.

$\frac{1}{4} \cdot 2 = \frac{1}{2}, \ \frac{1}{2} \cdot 2 = \frac{2}{2} = 1, \ 1 \cdot 2 = 2, \ 2 \cdot 2 = 4, \ 2 \cdot 4 = 8$

Pattern 2: Add $\frac{1}{4}$ to the previous term.

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

$\frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

Pattern 3: Multiply each term by half the reciprocal of the previous term.

$\frac{1}{4} \cdot \left(\frac{1}{2} \cdot 4\right) = \frac{1}{4} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

43. The value of y is 2 more than three times the value of x; $y = 3x + 2$;
Sample answer: If $x = 10$, then $y = 3(10) + 2 = 32$; If $x = 72$, then $y = 3(72) + 2 = 218$.

Chapter 2

44. a. Figure 1 has a perimeter of 4.
Figure 2 has a perimeter of 8.
Figure 3 has a perimeter of 12.
Figure 4 has a perimeter of 16.
Figure 5 has a perimeter of 20.
Figure 6 has a perimeter of 24.
Figure 7 has a perimeter of 28.
The perimeter is equal to the product of 4 and the figure number.

b. The 20th figure has a perimeter of $4(20) = 80$.

45. a. true; Based on the Law of Syllogism, if you went camping at Yellowstone, and Yellowstone is in Wyoming, then you went camping in Wyoming.

b. false; When you go camping, you go canoeing, but even though your friend always goes camping when you do, he or she may not choose to go canoeing with you.

c. true; We know that if you go on a hike, your friend goes with you, and we know that you went on a hike. So, based on the Law of Detachment, your friend went on a hike.

d. false; We know that you and your friend went on a hike, but we do not know where. We just know that there is a 3-mile long trail near where you went camping.

46. a. Mineral C must be Talc. Because it was scratched by all three of the other minerals, it must have the lowest hardness rating. Because Mineral B has a higher hardness rating than Mineral A, Mineral A could be either Gypsum or Calcite, and Mineral B could be either Calcite or Fluorite.

b. Check Mineral B and Mineral D. If Mineral D scratches Mineral B, then Mineral D is Fluorite, Mineral B is Calcite, and Mineral A is Gypsum. If Mineral B scratches Mineral D, then Mineral B is Fluorite, and you have to check Mineral D and Mineral A. The one that scratches the other has the higher hardness rating and is therefore Calcite. The one that gets scratched is Gypsum.

Maintaining Mathematical Proficiency

47. Segment Addition Postulate (Post. 1.2)

48. Angle Addition Postulate (Post. 1.4)

49. Ruler Postulate (Post. 1.1)

50. Protractor Postulate (Post. 1.3)

2.3 Explorations (p. 83)

1. The diagram can be turned at any angle to the right or to the left and the lines will appear to be perpendicular.

2. a. true; For every set of two intersecting lines, there is exactly one plane that is defined, so it can be assumed that all of the points shown are coplanar.

b. false; For every two points there is exactly one line, the third point does not necessarily have to be on the same line as the other two.

c. true; All three points lie on the same line, $\overleftrightarrow{AH}$.

d. true; $\angle GFH$ is marked as a right angle.

e. true; By definition of a linear pair, the sides of $\angle BCA$ and $\angle ACD$ form a straight line (straight angle).

f. false; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are not necessarily perpendicular because the angle is not marked.

g. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are not necessarily parallel, there is not enough information about the related angles.

h. true; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are coplanar.

i. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ could possibly intersect.

j. true; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ intersect at point C.

k. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{AH}$ are perpendicular. So, $\overleftrightarrow{EG}$ cannot be perpendicular to two different lines that intersect.

l. true; $\angle ACD$ and $\angle BCF$ form two pairs of opposite rays.

m. true; $\overleftrightarrow{AC}$ and $\overleftrightarrow{FH}$ are the same line because the points A, C, F, and H are all collinear.

3. You can assume intersecting lines, opposite rays, vertical angles, linear pairs, adjacent angles, coplanar (points, lines, rays, etc.), collinear points, which point is between two other points, and which points are in the interior of an angle. You have to have a label for identifying angle measures, segment lengths, perpendicular lines, parallel lines, and congruent segments or angles.

4. *Sample answer:* $\angle ACD$ and $\angle DCF$ for a linear pair, because these angles share a vertex and a side but no common interior points and $\angle ACF$ is a straight angle. $\angle CFE$ and $\angle GFH$ are vertical angles, because $\overleftrightarrow{FG}$ and $\overleftrightarrow{FE}$ are opposite rays as well as $\overleftrightarrow{FC}$ and $\overleftrightarrow{FH}$; $\angle DCF$ is a right angle, which cannot be assumed because angle measurements have to be marked. $\overline{BC} \cong \overline{CD}$, which cannot be assumed because lengths of segments have to be labeled.

2.3 Monitoring Progress (pp. 84–86)

1. Plane Intersection Postulate (Post. 2.7)

2. a. Line n passes through points A and B.

b. Line n contains points A and B.

c. Line m and line n intersect at point A.

3. Mark each segment with double tick marks to show that $\overline{PW} \cong \overline{WQ}$.

4. *Sample answer:* $\angle TWP$ and $\angle PWV$ are supplementary.

5. Yes, by the Plane Intersection Postulate (Post. 2.7), plane T intersects plane S at $\overleftrightarrow{BC}$.

6. Because of the right angle symbol you know that plane T is perpendicular to plane S. If $\overleftrightarrow{AB}$ is perpendicular to plane S and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{BC}$ in plane S at point B, then $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

Chapter 2

2.3 Exercises (pp. 87–88)

Vocabulary and Core Concept Check

1. Through any three noncollinear points, there exists exactly one plane.

2. Two points determine a line, which could be on infinitely many planes, but only one plane will go through those two points and a third noncollinear point.

Monitoring Progress and Modeling With Mathematics

3. Two Point Postulate (Post. 2.1): Through any two points there exists exactly one line.

4. Plane-Point Postulate (Post. 2.5): A plane contains at least three noncollinear points.

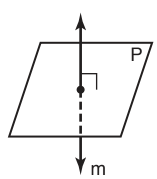
5. *Sample answer:* Line p contains points H and G.

6. *Sample answer:* Lines p and q intersect at point H.

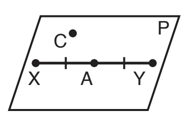
7. *Sample answer:* Through points J, G, and L there is exactly one plane, which is plane M.

8. *Sample answer:* Points J and K lie in plane M, so line q lies in plane M.

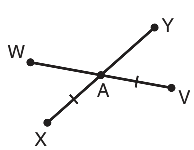
9. Plane P and line m intersect at a 90° angle.



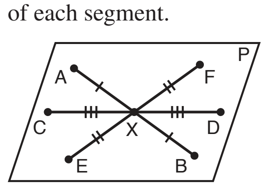
10. Plane P contains $\overline{XY}$, point A bisects $\overline{XY}$, and point C is not on $\overline{XY}$.



11. $\overline{XY}$ intersects $\overline{WV}$ at point A, so that $XA = VA$.



12. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all in plane P and point X is the midpoint of each segment.



13. Yes, planes W and X intersect at $\overleftrightarrow{KL}$.

14. Yes, N, K, and M are collinear with L not on the line, so these points are coplanar.

15. No, Q is a point contained in plane W, M is a point contained in plane X, and J is a point on the intersection of the planes, so they are three noncollinear points.

16. No, $\overleftrightarrow{RP}$ and $\overleftrightarrow{MN}$ both intersect $\overleftrightarrow{JL}$ (which is contained in both planes) at two different points.

17. Yes, the line of intersection is contained in both planes.

18. No, there is not enough information given.

19. Yes, $\angle NKL$ and $\angle JKM$ are vertical angles.

20. Yes, the nonadjacent sides form a straight angle.

21. In order to determine that M is the midpoint of $\overline{AC}$ or $\overline{BD}$, the segments that would have to be marked as congruent are $\overline{AM}$ and $\overline{MC}$ or $\overline{DM}$ and $\overline{MB}$, respectively; Based on the diagram and markings, you can assume $\overline{AC}$ and $\overline{DB}$ intersect at point M, such that $\overline{AM} \cong \overline{MB}$ and $\overline{DM} \cong \overline{MC}$.

22. In order to assume that an angle measures 90°, the angle must be marked as such; Based on the diagram, you can assume two pairs of vertical angles, $\angle DMC$ and $\angle AMB$ or $\angle DMA$ and $\angle CMB$, and you can assume linear pairs, such as $\angle DMC$ and $\angle CMB$.

23. The statements that cannot be concluded are: C, D, F, and H.

24. one; Based on the Line-Point Postulate (Post. 2.2), line m contains at least two points. Because these two points are noncollinear with point C, based on the Three Point Postulate (Post. 2.4), there is exactly one plane that goes through line m and point C.

25. Two Point Postulate (Post. 2.1)

26. Line Intersection Postulate (Post. 2.3)

27. Two Point Postulate: Through any two points, there exists exactly one line.

a. Conditional statement: If there are two points, then there exists exactly one line that passes through them.

b. Converse: If there exists exactly one line that passes through a given point or points, then there are two points. (False)

Inverse: If there are not two points, then there is not exactly one line that passes through them. (False)

Contrapositive: If there is not exactly one line that passes through a given point or points, then there are not two points. (True)

28. Plane-Point Postulate: A plane contains at least three noncollinear points.

a. Conditional statement: If a plane exists, then it contains at least three noncollinear points.

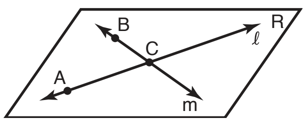
b. Converse: If a plane contains at least three noncollinear points, then the plane exists. (True)

Inverse: If no plane exists, then there are not three noncollinear points. (True)

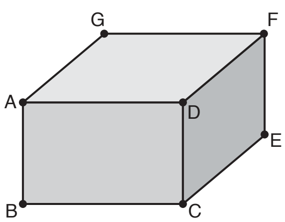
Contrapositive: If there are not three noncollinear points, then a plane has not been defined. (True)

29. Number of points to determine a line < number of points to determine a plane

30. yes; Let two lines ℓ and m intersect at point C. There must be a second point on each line, A in ℓ and B in m. Through the three noncollinear points A, B, and C, there exists exactly one plane R. Because A and C are in R, ℓ is in R. Because B and C are in R, m is in R.

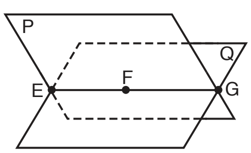


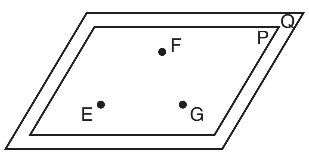
31. Yes, for example, three planes, *ABCD*, *DCEF*, and *DFGA*, have point D in common.



32. no; The postulate states that if two planes intersect, they will intersect in a line. But plans can be parallel and never intersect. For example, the ceiling and floor of a room are parallel.

33. Points E, F, and G must be collinear. They must be on the line that intersects plane P and plane Q; Points E, F, and G can be either collinear or not collinear.





34. *Sample answer:* The Line Intersection Postulate (Post. 2.3) would have to be altered. In spherical geometry, if two lines intersect, then their intersection is exactly two points. The two points of intersection would be the endpoints of a diameter.

Maintaining Mathematical Proficiency

35. Addition Property of Equality

$$t - 6 = -4$$
$$t - 6 + 6 = -4 + 6$$
$$t = 2$$

36. Division Property of Equality

$$3x = 21$$
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

37. Subtraction Property of Equality

$$9 + x = 13$$
$$9 - 9 + x = 13 - 9$$
$$x = 4$$

38. Multiplication Property of Equality

$$\frac{x}{7} = 5$$
$$7 \cdot \frac{x}{7} = 5 \cdot 7$$
$$x = 35$$

2.1–2.3 What Did You Learn? (p. 89)

1. "If you are in math class, then you are in geometry," is false. You could be in another math class, for example, you could be in Algebra I or Calculus.

"If you do your math homework, then you will do well on the test," is false. Some students can do all their homework, however, they may have test anxiety, in which case they may not do well on the test.

"If it does not snow, then I will run outside" is false. On a day that it is not snowing you may feel too sick to run outside.

2. a. p: You go to the zoo to see a lion.

q: You will see a cat.

p	q	q→p
T	T	T
T	F	F
F	T	T
F	F	T

b. p: You play a sport.

q: You wear a helmet.

p	q	q→p
T	T	T
T	F	F
F	T	T
F	F	T

c. p: This month has 31 days.

q: It is not February.

p	q	q→p
T	T	T
T	F	F
F	T	T
F	F	T

3. *Sample answer:* What about parallel lines? Do they intersect?

Chapter 2

2.1–2.3 Quiz (p. 90)

1. If-then form: If an angle measures 167°, then the angle is an obtuse angle. (True)

Converse: If an angle is obtuse, then the angle measures 167°. (False)

Inverse: If an angle does not measure 167°, then the angle is not an obtuse angle. (False)

Contrapositive: If an angle is not obtuse, then the measure of the angle is not 167°. (True)

2. If-then form: If you are in physics class, then you always have homework. (True)

Converse: If you always have homework, then you are in physics class. (False)

Inverse: If you are not in physics class, then you do not always have homework. (False)

Contrapositive: If you do not always have homework, then you are not in physics class. (True)

3. If-then form: If I take my driving test, then I will get my driver's license. (False)

Converse: If I get my driver's license, then I took my driving test. (True)

Inverse: If I do not take my driving test, then I will not get my driver's license. (True)

Contrapositive: If I do not get my driver's license, then I did not take my driving test. (False)

4. *Sample answer:* $5 + (-14) = -9$

5. *Sample answer:* A figure with four sides that is not a rectangle is a trapezoid.

6. The sum of two negative integers is a negative integer.

Inductive reasoning: $-2 + (-4) = -6$, $-23 + (-14) = -37$

Deductive reasoning: When you add two integers with the same sign, the rule is that you first add the absolute values, and then give the sum the same sign as the addends. So, the sum will be negative.

7. The difference of two even integers is an even integer.

Inductive reasoning: $4 - 2 = 2$, $84 - 62 = 22$

Deductive reasoning: Let m and n be integers. Then $2n$ and $2m$ are even integers because they are the product of 2 and an integer. $2n - 2m$ represents the difference of the two even integers. By the Distributive Property, $2n - 2m = 2(n - m)$, and $2(n - m)$ is an even integer because it is the product of 2 and an integer $(n - m)$.

8. Yes, points D, B, and C are coplanar, because three noncollinear points determine a plane.

9. No, in order for the planes to be parallel, it would have to be shown that the line that contains the points G, B, and A is perpendicular to each plane.

10. Yes, two lines intersect in one point.

11. Yes, if two points lie in the plane, then the line containing them lies in the plane.

12. No, there is no indication that $\overleftrightarrow{BG}$ is perpendicular to $\overrightarrow{BD}$.

13. no; The converse of that would be, "If I used the green ball, then I got a strike," and only one counterexample of using the green ball and not getting a strike or getting a strike with another color ball would be all you need to disprove the biconditional statement for the given conditional statement.

14. **a.** *Sample answer:* The males' running times are faster than the females' running times.

b. Inductive reasoning was used, because the conjecture was based on the specific cases represented in the table.

15. Two Point Postulate (Post. 2.1): Points C and D contain one line, $\overleftrightarrow{CD}$.

Line-Point Postulate (Post. 2.2): $\overleftrightarrow{CD}$ contains at least two points, C and D.

Line Intersection Postulate (Post. 2.3): Line m and line n intersect at exactly one point, G.

Three Point Postulate (Post. 2.4): Through points A, B, and C, there exists exactly one plane, X.

Plane-Point Postulate (Post. 2.5): Plane X contains the noncollinear points A, B, and D.

Plane-Line Postulate (Post. 2.6): Points A and B lie in plane X, therefore the line containing them, $\overleftrightarrow{AB}$, also lies in plane X.

Plane Intersection Postulate (Post. 2.7): Plane Y and plane X intersect in $\overleftrightarrow{CD}$.

2.4 Explorations (p. 91)

1. Distribution Property

Simplify.

Subtraction Property of Equality

Combine like terms.

Subtraction Property of Equality

Combine like terms.

Division Property of Equality

Simplify.

Symmetric Property of Equality

Chapter 2

2. The diamond represents multiplication because $0 \times 5 = 0$. The circle represents addition because $0 + 5 = 5$.
   Commutative Property of Multiplication
   Commutative Property of Addition
   Associative Property of Multiplication
   Associative Property of Addition
   Zero Property of Multiplication
   Identity Property of Addition
   Identity Property of Multiplication
   Distributive Property

3. Algebraic properties help you solve an equation by isolating the variable on one side of the equation.

4.

Equation	Reason
$3(x + 1) - 1 = -13$	Write the equation.
$3x + 3 - 1 = -13$	Distributive Property
$3x + 2 = -13$	Combine like terms.
$3x + 2 - 2 = -13 - 2$	Subtraction Property of Equality
$3x = -15$	Combine like terms.
$\dfrac{3x}{3} = \dfrac{-15}{3}$	Division Property of Equality
$x = -5$	Simplify.

2.4 Monitoring Progress (pp. 92–95)

1.

Equation	Explanation and Reason
$6x - 11 = -35$	Write the equation; Given
$6x - 11 + 11 = -35 + 11$	Add 11 to each side; Addition Property of Equality
$6x = -24$	Combine like terms; Simplify.
$x = -4$	Divide each side by 6; Division Property of Equality

2.

Equation	Explanation and Reason
$-2p - 9 = 10p - 17$	Write the equation; Given
$-2p - 10p - 9 = 10p - 10p - 17$	Subtract $10p$ from each side; Subtraction Property of Equality
$-12p - 9 = -17$	Combine like terms; Simplify.
$-12p - 9 + 9 = -17 + 9$	Add 9 to each side; Addition Property of Equality
$-12p = -8$	Combine like terms; Simplify.
$p = \frac{2}{3}$	Divide each side by -12; Division Property of Equality

3.

Equation	Explanation and Reason
$39 - 5z = -1 + 5z$	Write the equation; Given
$39 - 5z - 5z = -1 + 5z - 5z$	Subtract $5z$ from each side; Subtraction Property of Equality
$39 - 10z = -1$	Combine like terms; Simplify.
$39 - 39 - 10z = -1 - 39$	Subtract 39 from each side; Subtraction Property of Equality
$-10z = -40$	Combine like terms; Simplify.
$z = 4$	Divide each side by -10; Division Property of Equality

4.

Equation	Explanation and Reason
$3(3x + 14) = -3$	Write the equation; Given
$9x + 42 = -3$	Multiply; Distributive Property
$9x = -45$	Subtract 42 from each side; Subtraction Property of Equality
$x = -5$	Divide each side by 9; Division Property of Equality

5.

Equation	Explanation and Reason
$4 = -10b + 6(2 - b)$	Write the equation; Given
$4 = -10b + 12 - 6b$	Multiply; Distributive Property
$4 = -16b + 12$	Combine like terms; Simplify.
$-8 = -16b$	Subtract 12 from each side; Subtraction Property of Equality
$\frac{1}{2} = b$	Divide each side by -16; Division Property of Equality
$b = \frac{1}{2}$	Rewrite the equation; Symmetric Property of Equality

6.

Equation	Explanation and Reason
$A = \dfrac{1}{2}bh$	Write the equation; Given
$2A = bh$	Multiply each side by 2; Multiplication Property of Equality
$\dfrac{2A}{h} = b$	Divide each side by h; Division Property of Equality
$b = \dfrac{2A}{h}$	Rewrite the equation; Symmetric Property of Equality

$$b = \frac{2 \cdot 952}{56} = \frac{1904}{56} = 34$$

The base is 34 feet when the area is 952 square feet and the height is 56 feet.

7. The property illustrated is the Symmetric Property of Equality.

8. The property illustrated is the Reflexive Property of Equality.

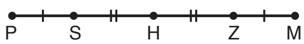
9. The property illustrated is the Transitive Property of Equality.

10. The property illustrated is the Transitive Property of Equality.

11. The property illustrated is the Symmetric Property of Equality.

12. The property illustrated is the Reflexive Property of Equality.

13.



Equation	Explanation and Reason
$SH = HZ$	Marked in diagram; Given
$PS = ZM$	Marked in diagram; Given
$PH = PS + SH$	Add lengths of adjacent segments; Segment Addition Postulate (Post. 1.2)
$HM = ZM + HZ$	Add lengths of adjacent segments; Segment Addition Postulate (Post. 1.2)
$PH = ZM + HZ$	Substitute ZM for PS and HZ for SH; Substitution Property of Equality
$PH = HM$	Substitute HM for $ZM + HZ$; Substitution Property of Equality

2.4 Exercises (pp. 96–98)

Vocabulary and Core Concept Check

1. Reflexive Property of Equality

2. "If $e = f$ and $f = g$, then $e = g$" is different. It represents the Transitive Property of Equality. The other three statements represent the Symmetric Property of Equality.

Monitoring Progress and Modeling with Mathematics

3. Subtraction Property of Equality

Addition Property of Equality

Division Property of Equality

4. Distributive Property

Subtraction Property of Equality

Addition Property of Equality

5.

Equation	Explanation and Reason
$5x - 10 = -40$	Write the equation; Given
$5x = -30$	Add 10 to each side; Addition Property of Equality
$x = -6$	Divide each side by 5; Division Property of Equality

6.

Equation	Explanation and Reason
$6x + 17 = -7$	Write the equation; Given
$6x = -24$	Subtract 17 from each side; Subtraction Property of Equality
$x = -4$	Divide each side by 6; Division Property of Equality

7.

Equation	Explanation and Reason
$2x - 8 = 6x - 20$	Write the equation; Given
$-4x - 8 = -20$	Subtract $6x$ from each side; Subtraction Property of Equality
$-4x = -12$	Add 8 to each side; Addition Property of Equality
$x = 3$	Divide each side by -4; Division Property of Equality

8.

Equation	Explanation and Reason
$4x + 9 = 16 - 3x$	Write the equation; Given
$7x + 9 = 16$	Add $3x$ to each side; Addition Property of Equality
$7x = 7$	Subtract 9 from each side; Subtraction Property of Equality
$x = 1$	Divide each side by 7; Division Property of Equality

9.

Equation	Explanation and Reason
$5(3x - 20) = -10$	Write the equation; Given
$15x - 100 = -10$	Multiply; Distributive Property
$15x = 90$	Add 100 to each side; Addition Property of Equality
$x = 6$	Divide each side by 15; Division Property of Equality

10.

Equation	Explanation and Reason
$3(2x + 11) = 9$	Write the equation; Given
$6x + 33 = 9$	Multiply; Distributive Property
$6x = -24$	Subtract 33 from each side; Subtraction Property of Equality
$x = -4$	Divide each side by 6; Division Property of Equality

11.

Equation	Explanation and Reason
$2(-x - 5) = 12$	Write the equation; Given
$-2x - 10 = 12$	Multiply; Distributive Property
$-2x = 22$	Add 10 to each side; Addition Property of Equality
$x = -11$	Divide each side by -2; Division Property of Equality

12.

Equation	Explanation and Reason
$44 - 2(3x + 4) = -18x$	Write the equation; Given
$44 - 6x - 8 = -18x$	Multiply; Distributive Property
$-6x + 36 = -18x$	Combine like terms; Simplify.
$36 = -12x$	Add $6x$ to each side; Addition Property of Equality
$-3 = x$	Divide each side by -12; Division Property of Equality
$x = -3$	Rewrite the solution; Symmetric Property of Equality

Chapter 2

13.

Equation	Explanation and Reason
$4(5x - 9) = -2(x + 7)$	Write the equation; Given
$20x - 36 = -2x - 14$	Multiply on each side; Distributive Property
$22x - 36 = -14$	Add $2x$ to each side; Addition Property of Equality
$22x = 22$	Add 36 to each side; Addition Property of Equality
$x = 1$	Divide each side by 22; Division Property of Equality

14.

Equation	Explanation and Reason
$3(4x + 7) = 5(3x + 3)$	Write the equation; Given
$12x + 21 = 15x + 15$	Multiply on each side; Distributive Property
$-3x + 21 = 15$	Subtract $15x$ from each side; Subtraction Property of Equality
$-3x = -6$	Subtract 21 from each side; Subtraction Property of Equality
$x = 2$	Divide each side by -3; Division Property of Equality

15.

Equation	Explanation and Reason
$5x + y = 18$	Write the equation; Given
$y = -5x + 18$	Subtract $5x$ from each side; Subtraction Property of Equality

16.

Equation	Explanation and Reason
$-4x + 2y = 8$	Write the equation; Given
$2y = 4x + 8$	Add $4x$ to each side; Addition Property of Equality
$y = 2x + 4$	Divide each side by 2; Division Property of Equality

17.

Equation	Explanation and Reason
$2y + 0.5x = 16$	Write the equation; Given
$2y = -0.5x + 16$	Subtract $0.5x$ from each side; Subtraction Property of Equality
$y = -0.25x + 8$	Divide each side by 2; Division Property of Equality

18.

Equation	Explanation and Reason
$\frac{1}{2}x - \frac{3}{4}y = -2$	Write the equation; Given
$-\frac{3}{4}y = -\frac{1}{2}x - 2$	Subtract $\frac{1}{2}x$ from each side; Subtraction Property of Equality
$y = \frac{2}{3}x + \frac{8}{3}$	Multiply each side by $-\frac{4}{3}$; Multiplication Property of Equality

19.

Equation	Explanation and Reason
$12 - 3y = 30x + 6$	Write the equation; Given
$-3y = 30x - 6$	Subtract 12 from each side; Subtraction Property of Equality
$y = -10x + 2$	Divide each side by -3; Division Property of Equality

20.

Equation	Explanation and Reason
$3x + 7 = -7 + 9y$	Write the equation; Given
$3x + 14 = 9y$	Add 7 to each side; Addition Property of Equality
$\frac{1}{3}x + \frac{14}{9} = y$	Divide each side by 9; Division Property of Equality
$y = \frac{1}{3}x + \frac{14}{9}$	Rewrite the equation; Symmetric Property of Equality

21.

Equation	Explanation and Reason
$C = 2\pi r$	Write the equation; Given
$\frac{C}{2\pi} = r$	Divide each side by 2π; Division Property of Equality
$r = \frac{C}{2\pi}$	Rewrite the equation; Symmetric Property of Equality

22.

Equation	Explanation and Reason
$I = Prt$	Write the equation; Given
$\frac{I}{rt} = P$	Divide each side by rt; Division Property of Equality
$P = \frac{I}{rt}$	Rewrite the equation; Symmetric Property of Equality

23.

Equation	Explanation and Reason
$S = 180(n - 2)$	Write the equation; Given
$\frac{S}{180} = n - 2$	Divide each side by 180; Division Property of Equality
$\frac{S}{180} + 2 = n$	Add 2 to each side; Addition Property of Equality
$n = \frac{S}{180} + 2$	Rewrite the equation; Symmetric Property of Equality

24.

Equation	Explanation and Reason
$S = 2\pi r^2 + 2\pi rh$	Write the equation; Given
$S - 2\pi r^2 = 2\pi rh$	Subtract $2\pi r^2$ from each side; Subtraction Property of Equality
$\frac{S - 2\pi r^2}{2\pi r} = h$	Divide each side by $2\pi r$; Division Property of Equality
$h = \frac{S - 2\pi r^2}{2\pi r}$	Rewrite the equation; Symmetric Property of Equality

25. The property illustrated is the Multiplication Property of Equality.

Chapter 2

26. The property illustrated is the Addition Property of Equality.

27. The property illustrated is the Reflexive Property of Equality.

28. The property illustrated is the Symmetric Property of Equality.

29. The property illustrated is the Reflexive Property of Equality.

30. The property illustrated is the Substitution Property of Equality.

31. The property illustrated is the Symmetric Property of Equality.

32. The property illustrated is the Transitive Property of Equality.

33. If $AB = 20$, then $AB + CD = 20 + CD$.

34. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$.

35. If $AB = CD$, then $AB + EF = CD + EF$.

36. If $AB = CD$, then $5 \bullet AB = 5 \bullet CD$.

37. If $LM = XY$, then $LM - GH = XY - GH$.

38. If $5(x + 8) = 2$, then $5x + 40 = 2$.

39. $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

40. $m\angle ABC = m\angle ABC$

41. The Subtraction Property of Equality should be used to subtract x from each side of the equation in order to get the second step.

$7x = x + 24$	Given
$6x = 24$	Subtraction Property of Equality
$x = 4$	Division Property of Equality

42. The reasons are wrong.

$6x + 14 = 32$	Given
$6x = 18$	Subtraction Property of Equality
$x = 3$	Division Property of Equality

43.

Equation	Explanation and Reason
$P = 2\ell + 2w$	Write the equation; Given
$P - 2w = 2\ell$	Subtract $2w$ from each side; Subtraction Property of Equality
$\frac{P - 2w}{2} = \frac{2\ell}{2}$	Divide each side by 2; Division Property of Equality
$\frac{P - 2w}{2} = \ell$	Simplify.
$\ell = \frac{P - 2w}{2}$	Rewrite the equation; Symmetric Property of Equality

$$\ell = \frac{32 - 2 \bullet 5}{2} = \frac{32 - 10}{2} = \frac{22}{2} = 11$$

The length is 11 meters.

44.

Equation	Explanation and Reason
$A = \frac{1}{2}h(b_1 + b_2)$	Write the equation; Given
$2 \bullet A = 2 \bullet \frac{1}{2}h(b_1 + b_2)$	Multiply each side by 2; Multiplication Property of Equality
$2A = h(b_1 + b_2)$	Simplify.
$2A = hb_1 + hb_2$	Multiply; Distributive Property
$2A - hb_2 = hb_1 + hb_2 - hb_2$	Subtract hb_2 from each side; Subtraction Property of Equality
$2A - hb_2 = hb_1$	Combine like terms; Simplify.
$\frac{2A - hb_2}{h} = \frac{hb_1}{h}$	Divide each side by h; Division Property of Equality
$\frac{2A - hb_2}{h} = b_1$	Simplify.
$b_1 = \frac{2A - hb_2}{h}$	Rewrite the equation; Symmetric Property of Equality

$$b_1 = \frac{2 \bullet 91 - 7 \bullet 20}{7} = \frac{182 - 140}{7} = \frac{42}{7} = 6$$

The other base is 6 meters.

45.

Equation	Explanation and Reason
$m\angle ABD = m\angle CBE$	Write the equation; Given
$m\angle ABD = m\angle 1 + m\angle 2$	Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)
$m\angle CBE = m\angle 2 + m\angle 3$	Add measures of adjacent angles; Angle Addition Postulate (Post. 1.4)
$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitute $m\angle 1 + m\angle 2$ for $m\angle ABD$; Substitution Property of Equality
$m\angle 1 = m\angle 3$	Subtract $m\angle 2$ from each side; Subtraction Property of Equality

46.

Equation	Explanation and Reason
$AC = BD$	Write the equation; Given
$AC = AB + BC$	Add measures of adjacent sides; Segment Addition Postulate (Post. 1.2)
$BD = BC + CD$	Add measures of adjacent sides; Segment Addition Postulate (Post. 1.2)
$AC = BC + CD$	Substitute AC for BD; Substitution Property of Equality
$AB + BC = BC + CD$	Substitute $AB + BC$ for AC; Substitution Property of Equality
$AB = CD$	Subtract BC from each side; Subtraction Property of Equality

Chapter 2

47.

Equation	Reason
$m\angle 1 = m\angle 4$, $m\angle EHF = 90°$, $m\angle GHF = 90°$	Given
$m\angle EHF = m\angle GHF$	Transitive Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Angle Addition Postulate (Post. 1.4)
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Transitive Property of Equality
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	Substitution Property of Equality
$m\angle 2 = m\angle 3$	Subtraction Property of Equality

48. Both properties state basic ideas about equality. The Reflexive Property of Equality states that something is equal to itself. So, both sides of the equal sign are identical. The Symmetric Property of Equality states that you can switch the two sides of an equation. So, two equations are equivalent if they have the same two expressions set equal to each other, but the expressions are on different sides of the equal sign.

49.

Equation	**Explanation and Reason**
$DC = BC, AD = AB$	Marked in diagram; Given
$AC = AC$	AC is equal to itself; Reflexive Property of Equality
$AC + AB + BC = AC + AB + BC$	Add $AB + BC$ to each side of $AC = AC$; Addition Property of Equality
$AC + AB + BC = AC + AD + DC$	Substitute AD for AB and DC for BC; Substitution Property of Equality

50.

Equation	**Explanation and Reason**
$BC = DA, CD = AB$	Marked in diagram; Given
$AC = AC$	AC is equal to itself; Reflexive Property of Equality
$AC + AB + BC = AC + AB + BC$	Add $AB + BC$ to each side of $AC = AC$; Addition Property of Equality
$AC + AB + BC = AC + CD + DA$	Substitute CD for AB and DA for BC; Substitution Property of Equality

51. $YX = 3, ZX = 5x + 17, YW = 10 - 2x$

$$ZX = ZY + YX$$
$$ZX = ZY + 3$$
$$ZY = ZX - 3$$

$$YW = YX + XW$$
$$YW = 3 + XW$$
$$XW = YW - 3$$

$$ZY = XW$$
$$ZX - 3 = YW - 3$$
$$(5x + 17) - 3 = (10 - 2x) - 3$$
$$5x + 14 = 7 - 2x$$
$$7x + 14 = 7$$
$$7x = -7$$
$$x = -1$$

$$ZY = 5x + 17 - 3 = 5(-1) + 17 - 3 = 9$$
$$XW = 10 - 2x - 3 = 10 - 2(-1) - 3 = 10 + 2 - 3 = 9$$

52. *Sample answer:* Reflexive: Employee 1 worked the same number of hours as Employee 1. Symmetric: If Employee 4 worked the same number of hours as Employee 5, then Employee 5 worked the same number of hours as Employee 4. Transitive: If Employee 2 worked the same number of hours as Employee 4, and Employee 4 worked the same number of hours as Employee 5, then Employee 2 worked the same number of hours as Employee 5.

53. The Symmetric Property of Equality is illustrated by A and B.

54. *Sample answer:* Reflexive: I earned the same number of points as myself on my favorite video game. This is reflexive because a quantity is equal to itself. Symmetric: If John had the same score as Tyeesha on our math quiz, then Tyeesha had the same score as John. This is Symmetric because the same two quantities are equal to each other. Transitive: If Dominic has the same number of pets as Ella, and Ella has the same number of pets as Brady, then Dominic has the same number of pets as Brady. This is transitive because the way we know that two quantities are equal is because they are each equal to a third quantity.

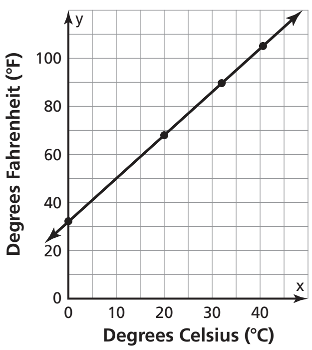
55. a.

Equation	**Explanation and Reason**
$C = \frac{5}{9}(F - 32)$	Write the equation; Given
$\frac{9}{5}C = F - 32$	Multiply each side by $\frac{9}{5}$; Multiplication Property of Equality
$\frac{9}{5}C + 32 = F$	Add 32 to each side; Addition Property of Equality
$F = \frac{9}{5}C + 32$	Rewrite the equation; Symmetric Property of Equality

b.

Degrees Celsius (°C)	$F = \frac{9}{5}C + 32$	Degrees Fahrenheit (°F)
0	$\frac{9}{5} \cdot 0 + 32 = 32$	32
20	$\frac{9}{5} \cdot 20 + 32 = 9 \cdot 4 + 32$ $= 68$	68
32	$\frac{9}{5} \cdot 32 + 32 = \frac{288}{5} + 32$ $= 57.6 + 32$ $= 89.6$	89.6
41	$\frac{9}{5} \cdot 41 + 32 = \frac{369}{5} + 32$ $= 73.8 + 32$ $= 105.8$	105.8

c. Yes, this is a linear function.



56. A, B, F; The Addition and Subtraction Properties are true because if you add (or subtract) the same amount to each side of an inequality, the inequality is still true. For the Substitution Property, two equal quantities could be substituted for each other in an inequality, but if one quantity is less than (or greater than) another quantity, you cannot always substitute one for the other into another inequality. The Reflexive Property is not true because quantities are not less than (or greater than) themselves. In order for the Symmetric Property to be true, the sign must be flipped around, as in if $a < b$, then $b > a$. The Transitive Property is true as long as all signs are going in the same direction. For example, if quantity A is less than quantity B, and quantity B is less than quantity C, then quantity A is less than quantity C.

Maintaining Mathematical Proficiency

57. Segment Addition Postulate (Post. 1.2)

58. Angle Bisector

59. Midpoint

60. Angle Addition Postulate (Post. 1.4)

2.5 Explorations (p. 99)

1. 2. Segment Addition Postulate (Post. 1.2)
3. Transitive Property of Equality
4. Subtraction Property of Equality

2. 1. $m\angle 1 = m\angle 3$
4. $m\angle 1 + m\angle 2$
5. $m\angle CBD$
6. $m\angle EBA = m\angle CBD$

3. You can use deductive reasoning to make statements about a given situation and use math definitions, postulates, and theorems as your reason or justification for each statement.

4. Given B is the midpoint of $\overline{AC}$.
C is the midpoint of $\overline{BD}$.
Prove $AB = CD$

STATEMENTS	REASONS
1. B is the midpoint of $\overline{AC}$. C is the midpoint of $\overline{BD}$.	1. Given
2. $\overline{AB} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$	2. Definition of midpoint
3. $AB = BC$, $BC = CD$	3. Definition of congruent segments
4. $AB = CD$	4. Transitive Property of Equality

2.5 Monitoring Progress (pp. 100–102)

1. Given T is the midpoint of $\overline{SU}$.
Prove $x = 5$

STATEMENTS	REASONS
1. T is the midpoint of $\overline{SU}$.	1. Given
2. $\overline{ST} \cong \overline{TU}$	2. Definition of midpoint
3. $ST = TU$	3. Definition of congruent segments
4. $7x = 3x + 20$	4. Substitution Property of Equality
5. $4x = 20$	5. Subtraction Property of Equality
6. $x = 5$	6. Division Property of Equality

Chapter 2

2. The property illustrated is the Reflexive Property of Segment Congruence (Thm. 2.1).

3. The property illustrated is the Symmetric Property of Angle Congruence (Thm. 2.2).

4. Step 5 would be $MB + MB = AB$.
Step 6 would be $2MB = AB$.
Step 7 would be $MB = \frac{1}{2}AB$.

2.5 Exercises (pp. 103–104)

Vocabulary and Core Concept Check

1. A postulate is a rule that is accepted to be true without proof and a theorem is a statement that can be proven by using definitions, postulates, and previously proven theorems.

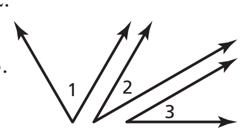
2. In a two column proof, each statement is on the left and each reason is on the right.

Monitoring Progress and Modeling with Mathematics

3. Given $PQ = RS$
Prove $PR = QS$

STATEMENTS	REASONS
1. $PQ = RS$	1. Given
2. $PQ + QR = RS + QR$	2. Addition Property of Equality
3. $PQ + QR = PR$	3. Segment Addition Postulate (Post. 1.2)
4. $RS + QR = QS$	4. Segment Addition Postulate (Post. 1.2)
5. $PR = QS$	5. Transitive Property of Equality

4. Given $\angle 1$ is a complement of $\angle 2$.
$\angle 2 \cong \angle 3$
Prove $\angle 1$ is a complement of $\angle 3$.



STATEMENTS	REASONS
1. $\angle 1$ is a complement of $\angle 2$.	1. Given
2. $\angle 2 \cong \angle 3$	2. Given
3. $m\angle 1 + m\angle 2 = 90°$	3. Definition of complementary angles
4. $m\angle 2 = m\angle 3$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 3 = 90°$	5. Substitution Property of Equality
6. $\angle 1$ is a complement of $\angle 3$.	6. Definition of complementary angles

5. Transitive Property of Segment Congruence (Thm. 2.1)

6. Reflexive Property of Angle Congruence (Thm. 2.2)

7. Symmetric Property of Angle Congruence (Thm. 2.2)

8. Reflexive Property of Segment Congruence (Thm. 2.1)

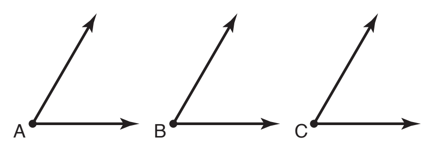
9. Symmetric Property of Segment Congruence (Thm. 2.1)

10. Transitive Property of Angle Congruence (Thm. 2.2)

11. Given Segment AB
Prove $\overline{AB} \cong \overline{AB}$

STATEMENTS	REASONS
1. A segment exists with endpoints A and B.	1. Given
2. AB equals the length of the segment with endpoints A and B.	2. Ruler Postulate (Post. 1.1)
3. $AB = AB$	3. Reflexive Property of Equality
4. $\overline{AB} \cong \overline{AB}$	4. Definition of congruent segments

12. Given $\angle A \cong \angle B$, $\angle B \cong \angle C$
Prove $\angle A \cong \angle C$

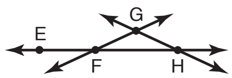


STATEMENTS	REASONS
1. $\angle A \cong \angle B$	1. Given
2. $m\angle A = m\angle B$	2. Definition of congruent angles
3. $\angle B \cong \angle C$	3. Given
4. $m\angle B = m\angle C$	4. Definition of congruent angles
5. $m\angle A = m\angle C$	5. Transitive Property of Equality
6. $\angle A \cong \angle C$	6. Definition of congruent angles

Chapter 2

13. Given $\angle GFH \cong \angle GHF$

Prove $\angle EFG$ and $\angle GHF$ are supplementary.

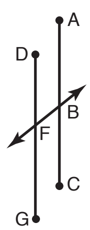


STATEMENTS	REASONS
1. $\angle GFH \cong \angle GHF$	1. Given
2. $m\angle GFH = m\angle GHF$	2. Definition of congruent angles
3. $\angle EFG$ and $\angle GFH$ form a linear pair.	3. Given (diagram)
4. $\angle EFG$ and $\angle GFH$ are supplementary.	4. Definition of linear pair
5. $m\angle EFG + m\angle GFH = 180°$	5. Definition of supplementary angles
6. $m\angle EFG + m\angle GHF = 180°$	6. Substitution Property of Equality
7. $\angle EFG$ and $\angle GHF$ are supplementary.	7. Definition of supplementary angles

14. Given $\overline{AB} \cong \overline{FG}$

$\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.

Prove $\overline{BC} \cong \overline{DF}$

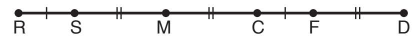


STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{FG}$	1. Given
2. $\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$.	2. Given
3. $\overline{BC} \cong \overline{AB}$, $\overline{FG} \cong \overline{DF}$	3. Definition of segment bisector
4. $\overline{BC} \cong \overline{FG}$	4. Transitive Property of Equality
5. $\overline{BC} \cong \overline{DF}$	5. Transitive Property of Segment Congruence (Thm. 2.1)

15. ~~The Transitive Property of Segment Congruence (Thm. 2.1) should have been used.~~ If $\overline{MN} \cong \overline{LQ}$ and $\overline{LQ} \cong \overline{PN}$, then $\overline{MN} \cong \overline{PN}$ by the Transitive Property of Segment Congruence (Thm. 2.1).

16. a. Given $\overline{RS} \cong \overline{CF}$, $\overline{SM} \cong \overline{MC} \cong \overline{FD}$

Prove $\overline{RM} \cong \overline{CD}$



b.

STATEMENTS	REASONS
1. $\overline{RS} \cong \overline{CF}$	1. Given
2. $RS = CF$	2. Definition of congruent segments
3. $\overline{SM} \cong \overline{FD}$	3. Given
4. $SM = FD$	4. Definition of congruent segments
5. $RM = RS + SM$	5. Segment Addition Postulate (Post. 1.2)
6. $CD = CF + FD$	6. Segment Addition Postulate (Post. 1.2)
7. $RS + SM = CD$	7. Substitution Property of Equality
8. $RM = CD$	8. Substitution Property of Equality
9. $\overline{RM} \cong \overline{CD}$	9. Definition of congruent segments

17. The triangle is an equiangular (or equilateral) triangle. By the Transitive Property of Angle Congruence (Thm. 2.2), because $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, you know that $\angle 1 \cong \angle 3$. Because all three angles are congruent, the triangle is equiangular. (It is also equilateral and acute.)

18. no; The statements have to have one segment in common in order to use the Transitive Property of Segment Congruence (Thm. 2.1), but in this case, the statements are about four different segments. They may or may not all be congruent to each other.

19. The purpose of a proof is to ensure the truth of a statement with such certainty that the theorem or rule proved could be used as a justification in proving another statement or theorem. Because inductive reasoning relies on observations about patterns in specific cases, the pattern may not continue or may change. So, the ideas cannot be used to prove ideas for the general case.

20. a. Given $\triangle JML$ is a right triangle.

Prove The acute angles of a right triangle are complementary.

b. Given $\triangle JML$ is a right triangle.

N is the midpoint of JM.

K is the midpoint of JL.

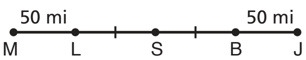
Prove $NK = \frac{1}{2}ML$

Chapter 2

21. a. It is a right angle.

b.

STATEMENTS	REASONS
1. $m\angle 1 + m\angle 1 + m\angle 2 + m\angle 2 = 180°$	1. Angle Addition Postulate (Post. 1.4)
2. $2(m\angle 1 + m\angle 2) = 180$	2. Distributive Property
3. $m\angle 1 + m\angle 2 = 90°$	3. Division Property of Equality

22. 

23. Given $\overline{QR} \cong \overline{PQ}, \overline{RS} \cong \overline{PQ}$

STATEMENTS	REASONS
1. $\overline{QR} \cong \overline{PQ}, \overline{RS} \cong \overline{PQ}$	1. Given
2. $QR = 2x + 5$, $RS = 10 - 3x$	2. Given
3. $QR = PQ$, $RS = PQ$	3. Definition of congruent segments
4. $QR = RS$	4. Transitive Property of Equality
5. $2x + 5 = 10 - 3x$	5. Substitution Property of Equality
6. $5x + 5 = 10$	6. Addition Property of Equality
7. $5x = 5$	7. Subtraction Property of Equality
8. $x = 1$	8. Division Property of Equality

Maintaining Mathematical Proficiency

24. $m\angle 1 + m\angle 4 = 90°$

$33° + m\angle 4 = 90°$

$m\angle 4 = 90° - 33°$

$m\angle 4 = 57°$

25. $m\angle 2 + m\angle 3 = 180°$

$147° + m\angle 3 = 180°$

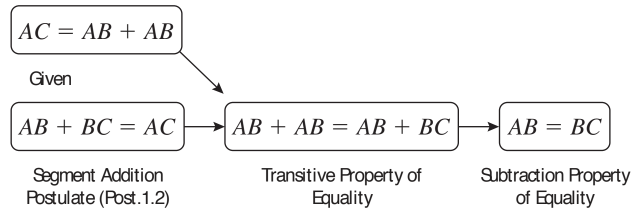
$m\angle 3 = 180° - 147°$

$m\angle 3 = 33°$

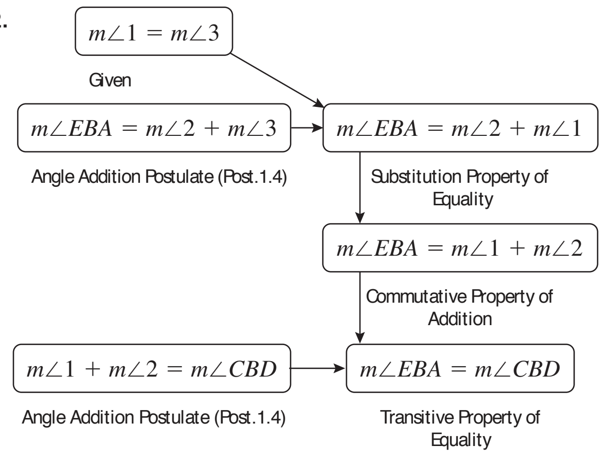
26. A pair of vertical angles are $\angle 1$ and $\angle 3$.

2.6 Explorations (p. 105)

1.



2.



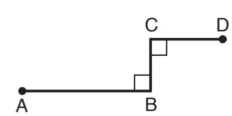
3. A flowchart uses boxes and arrows to show the flow of a logical argument.

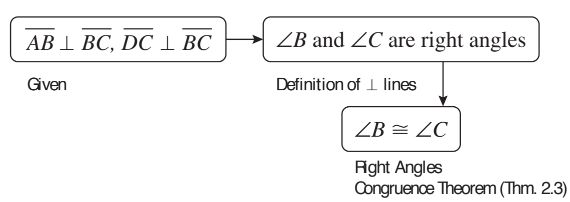
4. The flowchart proof, unlike the two-column proof, allows you to show explicitly which statement leads to which, but the two-column proof has a uniform, predictable shape and style and has each statement right below the previous one to allow for easy comparison. Both allow you to provide a logical argument and justification for why something is true.

2.6 Monitoring Progress (pp. 106–110)

1. Given $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$

Prove $\angle B \cong \angle C$



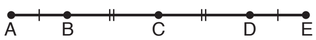


Chapter 2

STATEMENTS	REASONS
1. $\overline{AB} \perp \overline{BC}, \overline{DC} \perp \overline{BC}$	1. Given
2. $\angle B$ and $\angle C$ are right angles.	2. Definition of $\perp$ lines
3. $\angle B \cong \angle C$	3. Right Angles Congruence Theorem (Thm. 2.3)

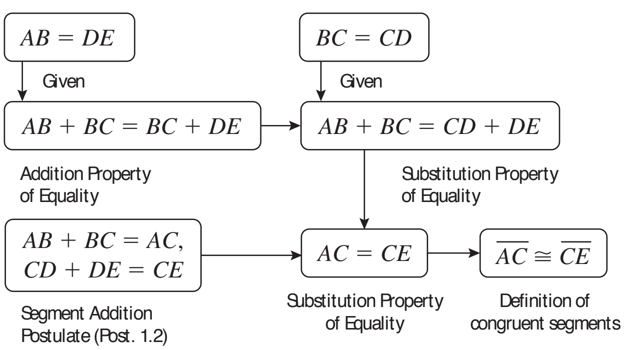
2. Given $AB = DE$, $BC = CD$

Prove $\overline{AC} \cong \overline{CE}$



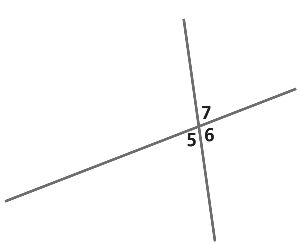
STATEMENTS	REASONS
1. $AB = DE$, $BC = CD$	1. Given
2. $AB + BC = BC + DE$	2. Addition Property of Equality
3. $AB + BC = CD + DE$	3. Substitution Property of Equality
4. $AB + BC = AC$, $CD + DE = CE$	4. Segment Addition Postulate (Post. 1.2)
5. $AC = CE$	5. Substitution Property of Equality
6. $\overline{AC} \cong \overline{CE}$	6. Definition of congruent segments

Flowchart proof:



3. Given $\angle 5$ and $\angle 7$ are vertical angles.

Prove $\angle 5 \cong \angle 7$



STATEMENTS	REASONS
1. $\angle 5$ and $\angle 7$ are vertical angles.	1. Given
2. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	2. Definition of linear pair
3. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	3. Linear Pair Postulate (Post. 2.8)
4. $m\angle 5 + m\angle 6 = 180°$, $m\angle 6 + m\angle 7 = 180°$	4. Definition of supplementary angles
5. $m\angle 6 + m\angle 7 = m\angle 5 + m\angle 6$	5. Transitive Property of Equality
6. $m\angle 5 = m\angle 7$	6. Subtraction Property of Equality
7. $\angle 5 \cong \angle 7$	7. Definition of congruent angles

By using the Congruent Supplement Theorem, you save three steps.

4. By the definition of supplementary angles, $m\angle 1 + m\angle 2 = 180°$.

$117° + m\angle 2 = 180°$

$m\angle 2 = 180° - 117° = 63°$

Vertical angles are congruent, so $\angle 1 \cong \angle 3$.

$m\angle 3 = 117°$

By the definition of supplementary angles, $m\angle 3 + m\angle 4 = 180°$.

$117° + m\angle 4 = 180°$

$m\angle 4 = 180° - 117° = 63°$

$m\angle 2 = 63°, m\angle 3 = 117°, m\angle 4 = 63°$

5. By the definition of supplementary angles, $m\angle 1 + m\angle 2 = 180°$.

$m\angle 1 + 59° = 180°$

$m\angle 1 = 180° - 59° = 121°$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

$m\angle 3 = 121°$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$. $m\angle 4 = 59°$

$m\angle 1 = 121°, m\angle 3 = 121°, m\angle 4 = 59°$

6. By the definition of supplementary angles, $m\angle 1 + m\angle 4 = 180°$.

$m\angle 1 + 88° = 180°$

$m\angle 1 = 180° - 88° = 92°$

Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.

$m\angle 2 = 88°$

Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.

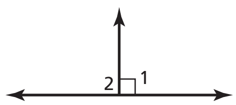
$m\angle 3 = 92°$

$m\angle 1 = 92°, m\angle 2 = 88°, m\angle 3 = 92°$

Chapter 2

7. $5w + 3 = 98$ Vertical angles are congruent.
$5w + 3 - 3 = 98 - 3$ Subtraction Property of Equality
$5w = 95$ Simplify.
$\frac{5w}{5} = \frac{95}{5}$ Division Property of Equality
$w = 19$ Simplify.

8. Given $\angle 1$ is a right angle.
Prove $\angle 2$ is a right angle.



$\angle 1$ is a right angle. By the definition of a right angle, $m\angle 1 = 90°$. $\angle 1$ and $\angle 2$ form a linear pair. So, by the Linear Pair Postulate (Post. 2.8), $\angle 1$ and $\angle 2$ are supplementary and $m\angle 1 + m\angle 2 = 180°$. By the Substitution Property of Equality, $90° + m\angle 2 = 180°$. Therefore, by the Subtraction Property of Equality, $m\angle 2 = 90°$. So, by definition, $\angle 2$ is a right angle.

2.6 Exercises (pp. 111–114)

Vocabulary and Core Concept Check

1. All right angles have a measure of 90°, and angles with the same measure are congruent.

2. Vertical angles and supplementary angles are formed by intersecting lines.

Monitoring Progress and Modeling with Mathematics

3. $\angle MSN \cong \angle PSQ$ by definition because they have the same measure; $\angle MSP \cong \angle PSR$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality; $\angle NSP \cong \angle QSR$ by the Congruent Complements Theorem (Thm. 2.5) because they are complementary to congruent angles.

4. $\angle FGH \cong \angle WXZ$, because $m\angle WXZ = 90°$ by the Angle Addition Postulate (Post. 1.4), which means that it is a right angle, and so, $\angle FGH$ and $\angle WXZ$ are congruent by the Right Angles Congruence Theorem (Thm. 2.3).

5. $\angle GML \cong \angle HMJ$ and $\angle GMH \cong \angle LMJ$ by the Vertical Angles Congruence Theorem (Thm. 2.6); $\angle GMK \cong \angle JMK$ by the Right Angles Congruence Theorem (Thm. 2.3). They form a linear pair, which means they are supplementary by the Linear Pair Postulate (Post. 2.8), and because one is a right angle, so is the other by the Subtraction Property of Equality.

6. $\angle ABC \cong \angle DEF$ by the Congruent Supplements Theorem (Thm. 2.4); $\angle CBD \cong \angle FEA$ by the Congruent Supplements Theorem (Thm. 2.4). $\angle DEF$ and $\angle FEA$ are supplementary, because they form a linear pair, and because $\angle CBD$ and $\angle FEA$ are supplementary to congruent angles, they are also congruent to each other.

7. Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.
$m\angle 3 = 143°$
By the definition of supplementary angles, $m\angle 1 + m\angle 2 = 180°$.
$143° + m\angle 2 = 180°$
$m\angle 2 = 180° - 143° = 37°$
Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.
$m\angle 4 = 37°$
$m\angle 2 = 37°, m\angle 3 = 143°, m\angle 4 = 37°$

8. Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.
$m\angle 1 = 159°$
By the definition of supplementary angles, $m\angle 2 + m\angle 3 = 180°$.
$m\angle 2 + 159° = 180°$
$m\angle 2 = 180° - 159° = 21°$
Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.
$m\angle 4 = 21°$
$m\angle 1 = 159°, m\angle 2 = 21°, m\angle 4 = 21°$

9. Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.
$m\angle 4 = 34°$
By the definition of supplementary angles, $m\angle 2 + m\angle 3 = 180°$.
$34° + m\angle 3 = 180°$
$m\angle 3 = 180° - 34° = 146°$
Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.
$m\angle 1 = 146°$
$m\angle 1 = 146°, m\angle 3 = 146°, m\angle 4 = 34°$

10. By the definition of supplementary angles, $m\angle 1 + m\angle 4 = 180°$.
$m\angle 1 + 29° = 180°$
$m\angle 1 = 180° - 29° = 151°$
Vertical angles are congruent. So, $\angle 2 \cong \angle 4$.
$m\angle 2 = 29°$
Vertical angles are congruent. So, $\angle 1 \cong \angle 3$.
$m\angle 3 = 151°$
$m\angle 1 = 151°, m\angle 2 = 29°, m\angle 3 = 151°$

Chapter 2

11.

$8x + 7 = 9x - 4$	Given
$8x + 7 - 8x = 9x - 4 - 8x$	Subtraction Property of Equality
$7 = x - 4$	Simplify.
$7 + 4 = x - 4 + 4$	Addition Property of Equality
$11 = x$	Simplify.

$5y = 7y - 34$	Given
$5y - 7y = 7y - 34 - 7y$	Addition Property of Equality
$-2y = -34$	Simplify.
$\dfrac{-2y}{-2} = \dfrac{-34}{-2}$	Division Property of Equality
$y = 17$	Simplify.

12.

$4x = 6x - 26$	Given
$4x - 6x = 6x - 26 - 6x$	Subtraction Property of Equality
$-2x = -26$	Simplify.
$\dfrac{-2x}{-2} = \dfrac{-26}{-2}$	Division Property of Equality
$x = 13$	Simplify.

$7y - 12 = 6y + 8$	Given
$7y - 12 - 6y = 6y + 8 - 6y$	Subtraction Property of Equality
$y - 12 = 8$	Simplify.
$y - 12 + 12 = 8 + 12$	Addition Property of Equality
$y = 20$	Simplify.

13.

$10x - 4 = 6(x + 2)$	Given
$10x - 4 = 6x + 12$	Distributive Property
$10x - 4 - 6x = 6x + 12 - 6x$	Subtraction Property of Equality
$4x - 4 = 12$	Simplify.
$4x - 4 + 4 = 12 + 4$	Addition Property of Equality
$4x = 16$	Simplify.
$\dfrac{4x}{4} = \dfrac{16}{4}$	Division Property of Equality
$x = 4$	Simplify.

$16y = 18y - 18$	Given
$16y - 18y = 18y - 18 - 18y$	Subtraction Property of Equality
$-2y = -18$	Simplify.
$\dfrac{-2y}{-2} = \dfrac{-18}{-2}$	Division Property of Equality
$y = 9$	Simplify.

14.

$2(5x - 5) = 6x + 50$	Given
$10x - 10 = 6x + 50$	Distributive Property
$10x - 10 - 6x = 6x + 50 - 6x$	Subtraction Property of Equality
$4x - 10 = 50$	Simplify.
$4x - 10 + 10 = 50 + 10$	Addition Property of Equality
$4x = 60$	Simplify.
$\dfrac{4x}{4} = \dfrac{60}{4}$	Division Property of Equality
$x = 15$	Simplify.

$5y + 5 = 7y - 9$	Given
$5y + 5 - 7y = 7y - 9 - 7y$	Subtraction Property of Equality
$-2y + 5 = -9$	Simplify.
$-2y + 5 - 5 = -9 - 5$	Subtraction Property of Equality
$-2y = -14$	Simplify.
$\dfrac{-2x}{-2} = \dfrac{-14}{-2}$	Division Property of Equality
$y = 7$	Simplify.

15. The expressions should have been set equal to each other because they represent vertical angles.

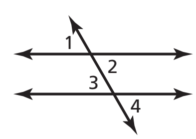
$$(13x + 45)^\circ = (19x + 3)^\circ$$
$$-6x + 45 = 3$$
$$-6x = -42$$
$$x = 7$$

16. Because the angles form a linear pair, the sum of their measures should be equal to 180°.

$$(13x + 45)^\circ + (12x - 40)^\circ = 180^\circ$$
$$25x + 5 = 180$$
$$25x = 175$$
$$\frac{25x}{25} = \frac{175}{25}$$
$$x = 7$$

17. Given $\angle 1 \cong \angle 3$

Prove $\angle 2 \cong \angle 4$



$\angle 1 \cong \angle 3$ → $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$ → $\angle 2 \cong \angle 3$ → $\angle 2 \cong \angle 4$

- $\angle 1 \cong \angle 3$: Given
- $\angle 1 \cong \angle 2,\ \angle 3 \cong \angle 4$: Vertical Angles Congruence Theorem (Thm. 2.6)
- $\angle 2 \cong \angle 3$: Transitive Property of Angle Congruence (Thm. 2.2)
- $\angle 2 \cong \angle 4$: Transitive Property of Angle Congruence (Thm. 2.2)

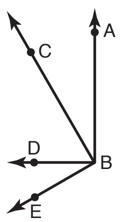
Chapter 2

Two column proof:

STATEMENTS	REASONS
1. $\angle 1 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Vertical Angles Congruence Theorem (Thm. 2.6)
3. $\angle 2 \cong \angle 3$	3. Transitive Property of Angle Congruence (Thm 2.2)
4. $\angle 2 \cong \angle 4$	4. Transitive Property of Angle Congruence (Thm 2.2)

18. Given $\angle ABD$ is a right angle.
$\angle CBE$ is a right angle.

Prove $\angle ABC \cong \angle DBE$



STATEMENTS	REASONS
1. $\angle ABD$ is a right angle. $\angle CBE$ is a right angle.	1. Given
2. $\angle ABC$ and $\angle CBD$ are complementary.	2. Definition of complementary angles
3. $\angle DBE$ and $\angle CBD$ are complementary.	3. Definition of complementary angles
4. $\angle ABC \cong \angle DBE$	4. Congruent Complements Theorem (Thm. 2.5)

Flowchart proof:

$\angle ABD$ is a right angle → $\angle ABC$ and $\angle CBD$ are complementary.

Given

Definition of complementary angles

$\angle CBE$ is a right angle → $\angle DBE$ and $\angle CBD$ are complementary.

Given

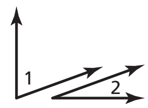
Definition of complementary angles

→ $\angle ABC \cong \angle DBE$

Congruent Complements Theorem (Thm. 2.5)

19. Given: $\angle 1$ and $\angle 2$ are complementary.
$\angle 1$ and $\angle 3$ are complementary.

Prove: $\angle 2 \cong \angle 3$



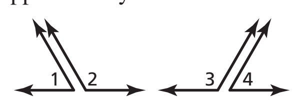
$\angle 1$ and $\angle 2$ are complementary, and $\angle 1$ and $\angle 3$ are complementary. By the definition of complementary angles, $m\angle 1 + m\angle 2 = 90°$ and $m\angle 1 + m\angle 3 = 90°$. By the Transitive Property of Equality, $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$. By the Subtraction Property of Equality, $m\angle 2 = m\angle 3$. So, $\angle 2 \cong \angle 3$ by the definition of congruent angles.

Two-column proof:

STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 3$ are complementary.	1. Given
2. $m\angle 1 + m\angle 2 = 90°$ $m\angle 1 + m\angle 3 = 90°$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 3$	3. Transitive Property of Equality
4. $m\angle 2 = m\angle 3$	4. Subtraction Property of Equality
5. $\angle 2 \cong \angle 3$	5. Definition of congruent angles

20. Given $\angle 1$ and $\angle 2$ are supplementary.
$\angle 3$ and $\angle 4$ are supplementary.
$\angle 1 \cong \angle 4$

Prove $\angle 2 \cong \angle 3$



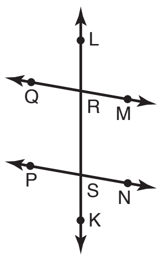
STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary. $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive Property of Equality
4. $m\angle 1 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5. Substitution Property of Equality
6. $m\angle 2 = m\angle 3$	6. Subtraction Property of Equality
7. $\angle 2 \cong \angle 3$	7. Definition of congruent angles

Paragraph proof:

Because $\angle 1$ and $\angle 2$ are supplementary and $\angle 3$ and $\angle 4$ are supplementary, $m\angle 1 + m\angle 2 = 180°$ and $m\angle 3 + m\angle 4 = 180°$ by the definition of supplementary angles. By the Transitive Property of Equality, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$. Because we are given that $\angle 1 \cong \angle 4$, by definition of congruent angles, $m\angle 1 = m\angle 4$. Therefore, by the Substitution Property of Equality, $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$, and by the Subtraction Property of Equality, $m\angle 2 = m\angle 3$. So, by definition of congruent angles, $\angle 2 \cong \angle 3$.

21. Given $\angle QRS$ and $\angle PSR$ are supplementary angles.

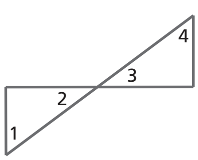
Prove $\angle QRL \cong \angle PSR$



Because $\angle QRS$ and $\angle PSR$ are supplementary, $m\angle QRS + m\angle PSR = 180°$ by the definition of supplementary angles. $\angle QRL$ and $\angle QRS$ form a linear pair and by definition are supplementary, which means that $m\angle QRL + m\angle QRS = 180°$. So, by the Transitive Property of Equality, $m\angle QRS + m\angle PSR = m\angle QRL + m\angle QRS$, and by the Subtraction Property of Equality, $m\angle PSR = m\angle QRL$. So, by definition of congruent angles, $\angle PSR \cong \angle QRL$, and by the Symmetric Property of Angle Congruence (Thm. 2.2), $\angle QRL \cong \angle PSR$.

22. Given $\angle 1$ and $\angle 3$ are complementary.
$\angle 2$ and $\angle 4$ are complementary.

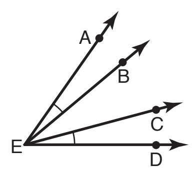
Prove $\angle 1 \cong \angle 4$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 3$ are complementary. $\angle 2$ and $\angle 4$ are complementary.	1. Given
2. $m\angle 1 + m\angle 3 = 90°$ $m\angle 2 + m\angle 4 = 90°$	2. Definition of complementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	3. Transitive Property of Equality
4. $\angle 2 \cong \angle 3$	4. Vertical Angles Congruence Theorem (Thm. 2.6)
5. $m\angle 2 = m\angle 3$	5. Definition of congruent angles
6. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 4$	6. Substitution Property of Equality
7. $m\angle 1 = m\angle 4$	7. Subtraction Property of Equality
8. $\angle 1 \cong \angle 4$	8. Definition of congruent angles

23. Given $\angle AEB \cong \angle DEC$

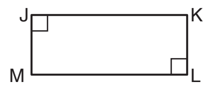
Prove $\angle AEC \cong \angle DEB$



STATEMENTS	REASONS
1. $\angle AEB \cong \angle DEC$	1. Given
2. $m\angle AEB = m\angle DEC$	2. Definition of congruent angles
3. $m\angle DEB = m\angle BEC + m\angle DEC$	3. Angle Addition Postulate (Post. 1.4)
4. $m\angle DEB = m\angle AEB + m\angle BEC$	4. Substitution Property of Equality
5. $m\angle AEC = m\angle BEC + m\angle AEB$	5. Angle Addition Postulate (Post. 1.4)
6. $m\angle AEC = m\angle DEB$	6. Transitive Property of Equality
7. $\angle AEC \cong \angle DEB$	7. Definition of congruent angles

Chapter 2

24. Given $\overline{JK} \perp \overline{JM}$, $\overline{KL} \perp \overline{ML}$, $\angle J \cong \angle M$, $\angle K \cong \angle L$
Prove $\overline{JM} \perp \overline{ML}$ and $\overline{JK} \perp \overline{KL}$

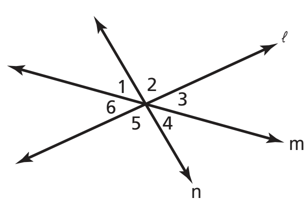


$\overline{JK} \perp \overline{JM}$
Given
$\angle J$ is a right angle
Definition of $\perp$ lines
$m\angle J = 90°$
Definition of right angle
$\angle J \cong \angle M$
Given
$m\angle J = m\angle M$
Definition of $\cong$ angles
$m\angle M = 90°$
Substitution Property of Equality
$\angle M$ is a right angle
Definition of right angle
$\overline{JM} \perp \overline{ML}$
Definition of $\perp$ lines

$\overline{KL} \perp \overline{ML}$
Given
$\angle L$ is a right angle
Definition of $\perp$ lines
$m\angle L = 90°$
Definition of right angle
$\angle K \cong \angle L$
Given
$m\angle K = m\angle L$
Definition of $\cong$ angles
$m\angle K = 90°$
Substitution Property of Equality
$\angle K$ is a right angle
Definition of right angle
$\overline{JK} \perp \overline{KL}$
Definition of $\perp$ lines

25. Your friend is correct. $\angle 1$ and $\angle 4$ are not vertical angles because they do not form two pairs of opposite rays. So, the Vertical Angles Congruence Theorem (Thm. 2.6) does not apply.

26.



If the measures of any two adjacent angles, such as $\angle 1$ and $\angle 2$ were given, then you could find the other four angle measures. In this case, you could find $m\angle 1 + m\angle 2$ and subtract this sum from 180° in order to find $m\angle 3$ (or $m\angle 6$). You can find the measures of the other three angles because each is a vertical angle with one of the three angles you know. Because vertical angles are congruent, $m\angle 4 = m\angle 1$, $m\angle 5 = m\angle 2$, and $m\angle 6 = m\angle 3$.

27. The converse statement is false: The converse is "If two angles are supplementary, then they are a linear pair." This is false because angles can be supplementary without being adjacent.

28. Time can be saved when writing proofs by using abbreviations and symbols instead of writing out the whole word. Also, when consecutive statements have the same reason, you can put them on the same line so that you only have to write the reason once.

29.
$$\begin{aligned}(7x + 4)° + (4x - 22)° &= 180° \\ 11x - 18 &= 180 \\ 11x &= 198 \\ x &= 18\end{aligned}$$

$$\begin{aligned}(3y + 11)° + 10y° &= 180° \\ 13y + 11 &= 180 \\ 13y &= 169 \\ y &= 13\end{aligned}$$

So, the angle measures are:
$10y° = 10(13) = 130°$
$(4x - 22)° = 4(18) - 22 = 50°$
$(7x + 4)° = 7(18) + 4 = 130°$
$(3y + 11)° = 3(13) + 11 = 50°$

30. **a.** The student is trying to prove that $\angle 1$ and $\angle 2$ are right angles by the definition of right angles.

b. No, because the last statement should be what is being proved: $\angle 1$ and $\angle 2$ are right angles. $\angle 1 \cong \angle 2$, was one of the given statements.

Maintaining Mathematical Proficiency

31. Three collinear points are E, J, H or B, I, C.

32. The intersection is $\overleftrightarrow{EF}$.

33. The two planes that contain $\overline{BC}$ are the planes containing any combination of three of the points A, B, C, D and any combination of three of the points B, C, G, F.

34. The three planes containing point D are the planes containing any combination of three of the points A, B, C, D, any combination of three of the points D, H, E, A and any combination of three of the points D, C, G, H.

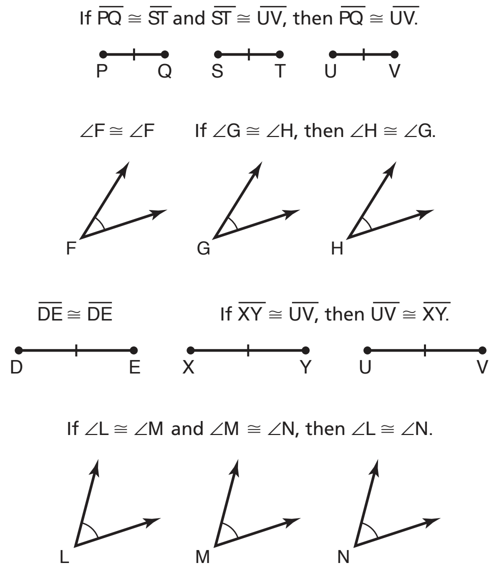
35. Three noncollinear points are: J, H, I (or any three points not on the same line).

36. The two planes containing J are the planes containing any combination of three of the points A, D, H, E and any combination of three of the points E, H, G, F.

2.4–2.6 What Did You Learn? (p. 115)

1. Even though the process for solving an equation may be almost automatic, when you have to justify each step, you have to think about the rules you are using and why you do each step in the process. When you think carefully about the rules and steps, you will make fewer mistakes, and this is how you know that your solution is a true statement for the given equation.

Chapter 2

2. 

3. $\overleftrightarrow{MQ}$ and $\overleftrightarrow{NP}$ could be meeting $\overleftrightarrow{LK}$ at different angles So, unless $\overleftrightarrow{MQ}$ and $\overleftrightarrow{NP}$ are parallel, $\angle QRL$ is not congruent to $\angle PSR$.

Chapter 2 Review (pp. 116–118)

1. Conditional statement: If two lines intersect, then their intersection is a point.

Converse: If two lines intersect in a point, then they are intersecting lines.

Inverse: If two lines do not intersect, then they do not intersect in a point.

Contrapositive: If two lines do not intersect in a point, then they are not intersecting lines.

Biconditional: Two lines intersect if and only if they intersect in a point.

2. Conditional: If $4x + 9 = 21$, then $x = 3$.

Converse: If $x = 3$, then $4x + 9 = 21$.

Inverse: If $4x + 9 \neq 21$, then $x \neq 3$.

Contrapositive: If $x \neq 3$, then $4x + 9 \neq 21$.

Biconditional: $4x + 9 = 21$ if and only if $x = 3$.

3. Conditional: If angles are supplementary, then they sum to 180°.

Converse: If angles sum to 180°, then they are supplementary.

Inverse: If angles are not supplementary, then they do not sum to 180°.

Contrapositive: If angles do not sum to 180°, then they are not supplementary.

Biconditional: Angles are supplementary if and only if they sum to 180°.

4. Conditional: If an angle is a right angle, then its measure is 90°.

Converse: If an angle measures 90°, then it is a right angle.

Inverse: If an angle is not a right angle, then its measure is not 90°.

Contrapositive: If an angle does not measure 90°, then it is not a right angle.

Biconditional: An angle is a right angle if and only if its measure is 90°.

5. Pattern: $5 - 3 = 2$, $17 - 13 = 4$

Conjecture: Odd integer − Odd integer = Even integer

Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1) - (2n + 1) &= 2m + 1 - 2n - 1\\ &= 2m - 2n\\ &= 2(m - n)\end{aligned}$$

Any number multiplied by 2 is an even number. So, the difference of any two odd integers is an even integer.

6. Pattern: $2 \cdot 3 = 6$, $4 \cdot 13 = 52$

Conjecture: Even integer × Odd integer = Even integer

Let m and n be integers. Then $2m$ is an even integer and $2n + 1$ is an odd integer.

$2m \cdot (2n + 1) = 2(2mn + m)$

Any number multiplied by 2 is an even number. So, the product of an even integer and an odd integer is an even integer.

7. $m\angle B = 90°$

8. If $4x = 12$, then $2x = 6$.

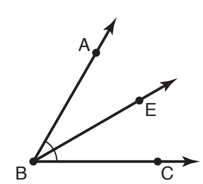
9. yes; Points A, B, C, and E are coplanar. $\overleftrightarrow{AB}$ and point C, which is not on $\overleftrightarrow{AB}$, lie in the same plane and point E, which is not on $\overleftrightarrow{AB}$ lie in the same plane.

10. yes; The right angle symbol indicates that $\overleftrightarrow{HC} \perp \overleftrightarrow{GE}$.

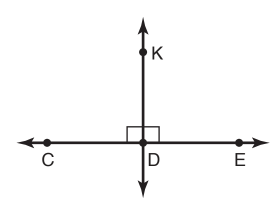
11. no; Points F, B, and G are not collinear.

12. no; There is not enough information to conclude that $\overleftrightarrow{AB} \parallel \overleftrightarrow{GE}$.

13. $\angle ABC$ is bisected by $\overrightarrow{BE}$.

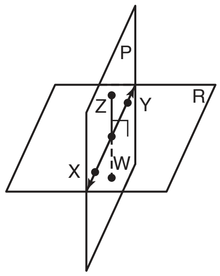


14. $\angle CDE$ is bisected by $\overleftrightarrow{DK}$.



Chapter 2

15. Plane $P \perp$ plane R and intersect in $\overleftrightarrow{XY}$, and $\overline{ZW}$ lies in plane P.



16.

Equation	**Explanation and Reason**
$-9x - 21 = -20x - 87$	Write the equation; Given
$11x - 21 = -87$	Add $20x$ to each side; Addition Property of Equality
$11x = -66$	Add 21 to each side; Addition Property of Equality
$x = -6$	Divide each side by 11; Division Property of Equality

17.

Equation	**Explanation and Reason**
$15x + 22 = 7x + 62$	Write the equation; Given
$8x + 22 = 62$	Subtract $7x$ from each side; Subtraction Property of Equality
$8x = 40$	Subtract 22 from each side; Subtraction Property of Equality
$x = 5$	Divide each side by 8; Division Property of Equality

18.

Equation	**Explanation and Reason**
$3(2x + 9) = 30$	Write the equation; Given
$6x + 27 = 30$	Multiply; Distributive Property
$6x = 3$	Subtract 27 from each side; Subtraction Property of Equality
$x = \frac{3}{6} = \frac{1}{2}$	Divide each side by 6; Division Property of Equality

19.

Equation	**Explanation and Reason**
$5x + 2(2x - 23) = -154$	Write the equation; Given
$5x + 4x - 46 = -154$	Multiply; Distributive Property
$9x - 46 = -154$	Combine like terms; Simplify.
$9x = -108$	Add 46 to each side; Addition Property of Equality
$x = -12$	Divide each side by 9; Division Property of Equality

20. Transitive Property of Equality

21. Reflexive Property of Equality

22. Symmetric Property of Angle Congruence (Thm. 2.2)

23. Reflexive Property of Angle Congruence (Thm 2.2)

24. Transitive Property of Equality

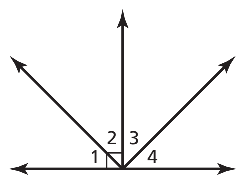
25. Given $\angle A$

Prove $\angle A \cong \angle A$

STATEMENTS	REASONS
1. An angle with vertex A exists.	1. Given
2. $m\angle A$ equals the measure of the angle with vertex A.	2. Protractor Postulate (Post. 1.3)
3. $m\angle A = m\angle A$	3. Reflexive Property of Equality
4. $\angle A \cong \angle A$	4. Definition of congruent angles

26. Given $\angle 3$ and $\angle 2$ are complementary.
$m\angle 1 + m\angle 2 = 90°$

Prove $\angle 3 \cong \angle 1$



STATEMENTS	REASONS
1. $m\angle 1 + m\angle 2 = 90°$	1. Given
2. $\angle 1$ and $\angle 2$ are complementary.	2. Definition of complementary angles
3. $\angle 3$ and $\angle 2$ are complementary.	3. Given
4. $\angle 3 \cong \angle 1$	4. Congruent Complements Theorem (Thm. 2.5)

Chapter 2 Test (p. 119)

1. no; No right angle is marked on $\overleftrightarrow{AB}$.

2. yes; Three noncollinear points determine a plane and all three points lie in plane P.

3. yes; Points E, C, and G all are on $\overleftrightarrow{GC}$.

4. yes; The intersection of two planes is a line by Postulate 2.7.

5. yes; The two points F and A lie in the same plane, so the line that contains them lies in the same plane.

6. no; $\overleftrightarrow{FG}$ is not drawn. So, you cannot be sure about where it intersects $\overleftrightarrow{AB}$.

Chapter 2

7.

Equation	Explanation and Reason
$9x + 31 = -23 + 3x$	Write the equation; Given
$6x + 31 = -23$	Subtract $3x$ from each side; Subtraction Property of Equality
$6x = -54$	Subtract 31 from each side; Subtraction Property of Equality
$x = -9$	Divide each side by 6; Division Property of Equality

8.

Equation	Explanation and Reason
$26 + 2(3x + 11) = -18$	Write the equation; Given
$26 + 6x + 22 = -18$	Multiply; Distributive Property
$6x + 48 = -18$	Combine like terms; Simplify.
$6x = -66$	Subtract 48 from each side; Subtraction Property of Equality
$x = -11$	Divide each side by 6; Division Property of Equality

9.

Equation	Explanation and Reason
$3(7x - 9) - 19x = -15$	Write the equation; Given
$21x - 27 - 19x = -15$	Multiply; Distributive Property
$2x - 27 = -15$	Combine like terms; Simplify.
$2x = 12$	Add 27 to each side; Addition Property of Equality
$x = 6$	Divide each side by 2; Division Property of Equality

10. Conditional: If two planes intersect, then their intersection is a line.

Converse: If two planes intersect in a line, then they are intersecting planes.

Inverse: If two planes do not intersect, then they do not intersect in a line.

Contrapositive: If two planes do not intersect in a line, then they are not intersecting planes.

Biconditional: Two planes intersect if and only if their intersection is a line.

11. Conditional: If an animal is a monkey, then the animal is a mammal.

Converse: If an animal is a mammal, then the animal is a monkey.

Inverse: If an animal is not a monkey, then the animal is not a mammal.

Contrapositive: If an animal is not a mammal, then the animal is not a monkey.

Biconditional: An animal is a monkey if and only if the animal is a mammal.

12. Pattern: $3 + 7 + 11 = 21, 5 + 13 + 15 = 33$

Conjecture: The sum of three odd integers is an odd integer.

Let ℓ, m, and n be integers. Then $(2\ell + 1)$, $(2m + 1)$, and $(2n + 1)$ are odd integers.

$$(2\ell + 1) + (2m + 1) + (2n + 1)$$
$$= 2\ell + 2m + 2n + 3$$
$$= 2(\ell + m + n + 1) + 1$$

The result is 1 more than an even integer (the product of 2 and $(1 + m + n + 1)$). So, the sum of three odd integers is an odd integer.

13. Pattern: $2 \cdot 4 \cdot 6 = 48$, $2 \cdot 10 \cdot 12 = 240$

Conjecture: The product of three even integers is a multiple of 8.

Let ℓ, m, and n be integers. Then 2ℓ, $2m$, and $2n$ represent even integers.

$(2\ell)(2m)(2n) = 8\ell mn$

The result, $8\ell mn$, is the product of 8 and integer ℓmn. So, the product is a multiple of 8.

14. *Sample answer:* If a figure is a rectangle, then it has four sides $ABCD$ has four sides.

15.

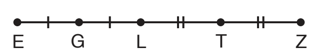
Equation	Explanation and Reason
$A = \frac{1}{2}bh$	Write the equation; Given
$2A = bh$	Multiply each side by 2; Multiplication Property of Equality
$\frac{2A}{b} = h$	Divide each side by b. Division Property of Equality

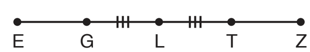
$$h = \frac{2 \cdot 558}{36} = \frac{1116}{36} = 31$$

The height of the sign is 31 inches.

16. Given: G is the midpoint of EL.
L is the midpoint of GT.
T is the midpoint of LZ.

Prove: $EG = TZ$





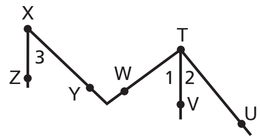
STATEMENTS	REASONS
1. G is the midpoint of EL. L is the midpoint of GT.	1. Given
2. $EG = GL$, $GL = LT$	2. Definition of midpoint
3. $EG = LT$	3. Transitive Property of Equality
4. T is the midpoint of LZ.	4. Given
5. $LT = TZ$	5. Definition of midpoint
6. $EG = TZ$	6. Transitive Property of Equality

Chapter 2

17. Given $\angle 2 \cong \angle 3$

$\overrightarrow{TV}$ bisects $\angle UTW$.

Prove $\angle 1 \cong \angle 3$



STATEMENTS	REASONS
1. $\angle 2 \cong \angle 3$	1. Given
2. $\overrightarrow{TV}$ bisects $\angle UTW$	2. Given
3. $\angle 1 \cong \angle 2$	3. Definition of angle bisector
4. $\angle 1 \cong \angle 3$	4. Transitive Property of Angle Congruence (Thm. 2.2)

Chapter 2 Standards Assessment (pp. 120–121)

1. a. Through points C and D, there exists exactly one line, $\overleftrightarrow{CD}$.

b. $\overleftrightarrow{AF}$ intersects $\overleftrightarrow{CB}$ at point B.

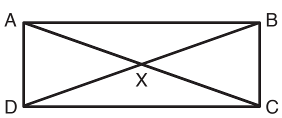
c. Through points E, B, and D, there exists exactly one plane S.

d. Points A and F lie in plane T, so $\overleftrightarrow{AF}$ also lies in plane T.

e. Planes T and S intersect in $\overleftrightarrow{CB}$.

2. Given $\overline{AX} \cong \overline{DX}$, $\overline{XB} \cong \overline{XC}$

Prove $\overline{AC} \cong \overline{BD}$



STATEMENTS	REASONS
1. $\overline{AX} \cong \overline{DX}$	1. Given
2. $AX = DX$	2. Definition of congruent segments
3. $\overline{XB} \cong \overline{XC}$	3. Given
4. $XB = XC$	4. Definition of congruent segments
5. $AX + XC = AC$	5. Segment Addition Postulate (Post. 1.2)
6. $DX + XB = DB$	6. Segment Addition Postulate (Post. 1.2)
7. $AC = DX + XB$	7. Substitution Property of Equality
8. $AC = BD$	8. Substitution Property of Equality
9. $\overline{AC} \cong \overline{BD}$	9. Definition of congruent segments

3. a. biconditional statement

b. inverse

c. converse

d. contrapositive

4. $AB = 3 + 1 = 4$

$BC = 6(4 - 3) = 6(1) = 6$

$CD = 4 \bullet 3 - 6 = 12 - 6 = 6$

$DE = 2(5 \bullet 3 - 7) - 8$

$= 2(15 - 7) - 8 = 2 \bullet 8 - 8 = 16 - 8 = 8$

$EF = 3(5 - 3) + 2 = 3(2) + 2 = 6 + 2 = 8$

$AB + BC + CD = 4 + 6 + 6 = 16$

$DE + EF = 8 + 8 = 16$

Segment BD is bisected by line □ Segment DF is bisected by line n. Segment AF is bisected by line m.

5. a. $\angle 3 \cong \angle 6$ by the Vertical Angles Congruence Theorem (Thm. 2.6).

b. $m\angle 4 \cong m\angle 7$ by the Vertical Angles Congruence Theorem (Thm. 2.6).

c. $m\angle FHE \neq m\angle AHG$

d. $m\angle AHG + m\angle GHE = 180°$ by the Linear Pair Postulate (Post. 2.8).

6. a. $AB = \sqrt{(-1 - (-6))^2 + (6 - 1)^2}$

$= \sqrt{(-1 + 6)^2 + (6 - 1)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25 + 25} = \sqrt{50} \approx 7.07$

b. $CD = \sqrt{(5 - (-5))^2 + (8 - 8)^2}$

$= \sqrt{(5 + 5)^2 + 0^2}$

$= \sqrt{(10)^2} = \sqrt{100} = 10$

c. $EF = \sqrt{(4 - 2)^2 + (-2 - 7)^2}$

$= \sqrt{2^2 + (-9)^2}$

$= \sqrt{4 + 81} = \sqrt{85} \approx 9.22$

d. $GH = \sqrt{(7 - 7)^2 + (-1 - 3)^2}$

$= \sqrt{0^2 + (-4)^2}$

$= \sqrt{16} = 4$

e. $JK = \sqrt{(1 - (-4))^2 + (-5 - (-2))^2}$

$= \sqrt{(1 + 4)^2 + (-5 + 2)^2}$

$= \sqrt{5^2 + (-3)^2}$

$= \sqrt{25 + 9} = \sqrt{34} \approx 5.83$

f. $LM = \sqrt{(7 - 3)^2 + (-5 - (-8))^2}$

$= \sqrt{(4)^2 + (-5 + 8)^2}$

$= \sqrt{4^2 + (-3)^2}$

$= \sqrt{16 + 9} = \sqrt{25} = 5$

The line segments in order from longest to shortest are $\overline{CD}$, $\overline{EF}$, $\overline{AB}$, $\overline{JK}$, $\overline{LM}$, and $\overline{GH}$.

7. $\angle PSK$ and $\angle NSR$ are vertical angles. So, by the Vertical Angles Congruence Theorem (Thm. 2.6), $\angle PSK \cong \angle NSR$. $\angle MRL$ and $\angle QRS$ are vertical angles. So, by the Vertical Angles Congruence Theorem (Thm. 2.6), $\angle MRL \cong \angle QRS$. Because $\angle MRL \cong \angle NSR$, you can conclude by the Transitive Property of Angle Congruence (Thm. 2.2) that $\angle QRS \cong \angle NSR$. So, the other angles that are also congruent to $\angle NSR$ are $\angle PSK$ and $\angle QRS$.

8. no; In order to prove the Vertical Angles Congruence Theorem (Thm. 2.6), you must state that $\angle 1$ and $\angle 3$ each form a linear pair with $\angle 2$, and therefore each is supplementary to $\angle 2$ by the Linear Pair Postulate (Post. 2.8). You can then state that $\angle 1$ and $\angle 3$ are congruent by the Congruent Supplements Theorem (Thm. 2.4).