Chapter 11

Chapter 11 Maintaining Mathematical Proficiency (p. 591)

1. \( S = 2\ell w + 2\ell h + 2wh \)
   \[ = 2(5)(8) + 2(5)(3) + 2(3)(8) \]
   \[ = 80 + 30 + 48 \]
   \[ = 158 \]
   The surface area is 158 square feet.

2. \( S = 2\left(\frac{1}{2}bh\right) + \ell h + wh + \ell w \)
   \[ = 2\left(\frac{1}{2} \cdot 6 \cdot 8\right) + 8(4) + 4(10) + 6(4) \]
   \[ = 48 + 32 + 40 + 24 \]
   \[ = 144 \]
   The surface area is 144 square meters.

3. \( S = 2\left(\frac{1}{2}bh\right) + 2\ell h + wh \)
   \[ = 2\left(\frac{1}{2} \cdot 4 \cdot 6\right) + 2(5)(10) + 6(10) \]
   \[ = 24 + 100 + 60 \]
   \[ = 184 \]
   The surface area is 184 square centimeters.

4. \( P = 2(\ell + w) \)
5. \( A = \frac{1}{2}bh \)
   \[ 28 = 2(\ell + 5) \]
   \[ 28 = 2\ell + 10 \]
   \[ 18 = 2\ell \]
   \[ 9 = \ell \]
   The length is 9 inches.

6. \( A = \ell w \)
   \[ 84 = 7\ell \]
   \[ 12 = \ell \]
   The length is 12 feet.

7. \( S = 2\ell w + 2hw + 2\ell h \)
   \[ = 2(x)(x) + 2(x)(x) + 2(x)(x) \]
   \[ = 2x^2 + 2x^2 + 2x^2 \]
   \[ = 6x^2 \]
   The prism is a cube.

Chapter 11 Mathematical Practices (p. 592)

1.

2.

3.

11.1 Explorations (p. 593)

1. a. The arc length is the circumference, which is \( C = 8\pi \approx 25.13 \) units.
   b. The arc length is \( \frac{1}{4} \cdot 8\pi = 2\pi \approx 6.28 \) units.
   c. The arc length is \( \frac{3}{5} \cdot 10\pi = \frac{6}{5}\pi \approx 10.47 \) units.
   d. The arc length is \( \frac{5}{6} \cdot 6\pi = \frac{15}{4}\pi \approx 11.78 \) units.

2. No; The additional tire revolution is about \( \frac{1}{2} \cdot 25\pi \approx 39.27 \) inches, or 3.27 feet. Since the mat is 3 feet, and 3 < 3.27, the tire will be off the mat.

3. To determine the length of an arc, multiply the fraction of the circle the arc represents by the circumference of the circle.

4. The front tire makes a revolution of about \( \frac{3}{4} \cdot 24\pi = 18\pi \approx 56.55 \) inches.

11.1 Monitoring Progress (pp. 594–597)

1. \( C = \pi d \)
   \[ = \pi \cdot 5 \]
   \[ \approx 15.71 \]
   The circumference is about 15.71 inches.

2. \( C = \pi d \)
   \[ 17 = \pi d \]
   \[ \frac{17}{\pi} = d \]
   \[ 5.41 \approx d \]
   The diameter is about 5.41 feet.

3. Arc length of \( PQ = \frac{75\degree}{360\degree} \cdot 9\pi \)
   \[ = \frac{5}{25} \cdot 9\pi \]
   \[ \approx 5.89 \]
   The arc length of \( PQ \) is about 5.89 yards.
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4. \[ \text{Arc length of } LM = \frac{mLM}{360^\circ} \]
   \[ \frac{61.26}{C} = \frac{270^\circ}{360^\circ} \]
   \[ \frac{61.26}{C} = \frac{3}{4} \]
   \[ \frac{4}{3} \cdot 61.26 = C \]
   \[ 81.68 = C \]
   The circumference is 81.68 meters.

5. \[ \text{Arc length of } EF = \frac{mEF}{360^\circ} \cdot 2\pi \]
   \[ 10.5 = \frac{150^\circ}{360^\circ} \cdot 2\pi \]
   \[ 10.5 = \frac{5}{12} \cdot 2\pi \]
   \[ 12 \cdot 10.5 \cdot \pi = r \]
   \[ 40.1 = r \]
   The radius is approximately 4.01 feet.

6. The circumference of the tire is \(28\pi\) inches. So, the tire travels \(28\pi\) inches, or \(\frac{7}{3}\) feet in one revolution. While traveling 500 feet, the tire makes about \(500 \div \frac{7}{3} \approx 68\) revolutions.

7. Distance = 2 \cdot \text{length of each straight section} + 2 \cdot \text{length of each semicircle}
   \[ = 2 \cdot 84.39 + 2\left(\frac{1}{2} \cdot 2 \cdot 44.02 \cdot \pi\right) \]
   \[ = 168.89 + 276.59 \]
   \[ = 445.48 \]
   The runner on the blue path travels about 445.48 meters.

8. \[ 15^\circ \left(\frac{\pi}{180^\circ}\right) = \frac{15\pi}{180} = \frac{\pi}{12} \text{ radian} \]

9. \[ \left(\frac{4\pi}{3}\right) \cdot \left(\frac{180}{\pi}\right) = 4 \cdot 60 = 240^\circ \]

11.1 Exercises (pp. 598–600)

Vocabulary and Core Concept Check

1. The circumference of a circle with diameter \(d\) is \(C = \pi d\).

2. Arc measure is measured in degrees and is based upon angles formed by the central angle or intercepted arcs. The length of an arc is a fractional part of the circumference of the circle.

Monitoring Progress and Modeling with Mathematics

3. \[ C = 2\pi \]
   \[ = 2 \cdot \pi \cdot 6 \]
   \[ = 12\pi \approx 37.70 \]
   The circumference of a circle with a radius 6 inches is about 37.70 inches.

4. \[ C = \pi d \]
   \[ 63 = \pi d \]
   \[ \frac{63}{\pi} = d \]
   \[ 20.05 = d \]
   The diameter of a circle with a circumference of 63 feet is about 20.05 feet.

5. \[ C = 2\pi \]
   \[ 28\pi = 2\pi \]
   \[ \frac{28\pi}{2\pi} = r \]
   \[ 14 = r \]
   The radius of a circle with a circumference of 28\pi units is 14 units.

6. \[ C = \pi d \]
   \[ C = 5\pi \]
   The exact circumference of a circle with a diameter of 5 inches is 5\pi inches.

7. \[ \text{Arc length of } AB = \frac{45^\circ}{360^\circ} \cdot 8\pi \]
   \[ = \frac{1}{8} \cdot 8\pi \]
   \[ = \pi \]
   \[ \approx 3.14 \]
   The arc length of \(AB\) is about 3.14 feet.

8. \[ \text{Arc length of } DE = \frac{mDE}{360^\circ} \cdot 2\pi \]
   \[ = \frac{8.73}{360^\circ} \cdot 2 \cdot \pi \cdot 10 \]
   \[ 360^\circ \cdot \frac{8.73}{20\pi} = mDE \]
   \[ \approx 50.02 \approx mDE \]
   The measure of \(DE\) is about 50.02°.

9. \[ \text{Arc length of } FG = \frac{mFG}{360^\circ} \cdot C \]
   \[ \frac{7.5}{\frac{76^\circ}{360^\circ}} = C \]
   \[ 35.53 \approx C \]
   The circumference of \(\odot C\) is about 35.53 meters.

10. \[ \text{Arc length of } LM = \frac{mLM}{360^\circ} \cdot 2\pi \]
    \[ 38.95 = \frac{260^\circ}{360^\circ} \cdot 2\pi \]
    \[ 38.95 = \frac{13}{9}\pi \]
    \[ \frac{9 \cdot 38.95}{13\pi} = r \]
    \[ 8.58 = r \]
    The radius of \(\odot R\) is about 8.58 centimeters.
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11. The value of the diameter was used for the radius.
   \[ C = \pi d \]
   \[ = 9\pi \text{ in.} \]

12. The arc measure should be divided by 360°.
   \[ \text{Arc length of } GH = \frac{mGH}{360°} \cdot 2\pi \]
   \[ = \frac{75°}{360°} \cdot 2\pi \cdot 5 \]
   \[ = \frac{25}{12\pi} \text{ cm} \]

13. The circumference of the wheel is \( 8\pi \) or about 25.13 inches.
   The length of the path is 25.13 \( \cdot \) 87 = 2186.31 inches.
   Therefore, the length of the path to the nearest foot is
   2186.31 \div 12 \approx 182 \text{ feet.} \]

14. The circumference of the front wheel is \( 2 \cdot \pi \cdot 32.5 = 65\pi \) centimeters.
   So, the wheel travels \( 65\pi \) centimeters, or 0.65\pi meters in one revolution.
   When you ride your bicycle 40 meters, the front wheel makes about
   \( 40 \div 0.65\pi \approx 20 \) revolutions.

15. The two horizontal edges are 2 \( \cdot \) 13 = 26 units.
   The circumference of the two semicircles is about
   \( 2 \cdot \frac{1}{2} \cdot \pi \cdot 6 = 6\pi \approx 18.85 \) units.
   The perimeter of the shaded region is about
   26 + 18.85 = 44.85 units.

16. The two horizontal edges are 2 \( \cdot \) 6 = 12 units.
   The arc length of the two arcs of the circles is about
   \( 2 \cdot \frac{90°}{360°} \cdot 2 \cdot \pi \cdot 3 \approx 9.42 \) units.
   The perimeter of the shaded region is about
   12 + 9.42 = 21.42 units.

17. The four straight edges are 4 \( \cdot \) 2 = 8 units.
   The four arc lengths are
   about 4 \( \cdot \frac{90°}{360°} \cdot 2 \cdot \pi \cdot 2 = 4\pi \approx 12.57 \) units.
   The perimeter of the shaded region is about
   8 + 12.57 = 20.57 units.

18. The radius of the circles is half the distance between the centers.
   Therefore, the radius of each circle is 2.5 units.
   The three arc lengths are
   \( 3 \cdot \frac{120°}{360°} \cdot 2 \cdot \pi \cdot 2.5 = 5\pi \approx 15.71 \) units.
   The three straight sections are 3 \( \cdot \) 5 = 15 units.
   The perimeter of the shaded region is about
   15.71 + 15 = 30.71 units.

19. \( 70° \cdot \left( \frac{\pi}{180°} \right) = \frac{70\pi}{180°} = \frac{7\pi}{18} \text{ radian} \)

20. \( 300° \cdot \left( \frac{\pi}{180°} \right) = \frac{300\pi}{180°} = \frac{5\pi}{3} \text{ radian} \)

21. \( \left( \frac{11\pi}{12} \right) \cdot \left( \frac{180°}{\pi} \right) = 11 \cdot 15° = 165° \)

22. \( \left( \frac{\pi}{8} \right) \cdot \left( \frac{180°}{\pi} \right) = \frac{45°}{2} = 22.5° \)

23. One revolution of the Ferris wheel is about
   \( 2 \cdot \pi \cdot 67.5 \approx 424.1 \) meters. The Ferris wheel travels
   about 0.26 meter per second, so one revolution is
   approximately \( \frac{424.1}{0.26} = 1631.2 \) seconds.
   For the Ferris wheel to travel one revolution, it takes about
   1631.2 seconds or 27.2 minutes.

24. A, B;
   \[ C = 2\pi r \]
   \[ \frac{4}{3} \cdot 38 = 2\pi r \]
   \[ \frac{4}{3} \cdot 38 \]
   \[ 5.26 \approx r \]
   The maximum radius is about 8.06 feet.

25. The radius of circle \( x^2 + y^2 = 16 \) is 4.
   \[ C = 2\pi r \]
   \[ = 2 \cdot \pi \cdot 4 = 8\pi \]
   The circumference of the circle with the equation
   \( x^2 + y^2 = 16 \) is \( 8\pi \) units.

26. The radius of circle \((x + 2)^2 + (y - 3)^2 = 9\) is 3.
   \[ C = 2\pi r \]
   \[ = 2 \cdot \pi \cdot 3 = 6\pi \]
   The circumference of the circle with the equation
   \((x + 2)^2 + (y - 3)^2 = 9\) is \( 6\pi \) units.

27. The center of the circle whose diameter has the endpoints
   \((-2, 5)\) and \((2, 8)\) is the midpoint.
   \[ 
   \text{Center} = \left( -\frac{2 + 2}{2}, \frac{5 + 8}{2} \right) = (0, 6.5) 
   \]
   \[ r^2 = (0 - 2)^2 + (6.5 - 8)^2 
   \]
   \[ r = \sqrt{(-2)^2 + (-1.5)^2} 
   \]
   \[ r = \sqrt{4 + 2.25} = 2.5 \]
   The arc length is \( \frac{180°}{360°} \cdot 2 \cdot \pi \cdot 2.5 \)
   \[ = 2.5 \cdot \pi \approx 7.85 \]
   The arc length of the semicircle is about 7.85 units.

28. a. Original: Arc length \( \overline{EF} = \frac{x}{360°} \cdot 2\pi \)
   
   Doubled radius: Arc length \( \overline{EF} = \frac{x}{360°} \cdot 2\pi(2r) \)
   \[ = 2 \left( \frac{x}{360°} \cdot 2\pi \right) \]
   By factoring a 2, the arc length is 2 times the original arc length \( \overline{EF} \).
   Therefore, when the radius is doubled, the arc length is doubled.
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34. a. The hypotenuse of the right triangle is the diameter: 
\[ h = \sqrt{12^2 + 16^2} = 20 \text{ inches}. \] Therefore, the circumference is about \( 20 \cdot \pi \approx 62.83 \) inches.

b. The diagonal of the square is the diameter. So, the diameter is \( 6\sqrt{2} \) centimeters and the radius is \( 3\sqrt{2} \) centimeters. Therefore, the circumference is about \( 6\sqrt{2}\pi \approx 26.66 \) inches.

c. The center of the circle to one side of the triangle is the radius of the inscribed circle.

[Diagram of a right triangle with sides labeled 4.5 inches, 3 inches, and hypotenuse labeled 9 inches, with radius labeled r and angle labeled 30°.]

The radius is opposite the 30° angle and 4.5 is opposite the 60° angle. Therefore, \( r = 4.5\sqrt{3} \approx 7.699 \) inches and the radius is \( \frac{3\sqrt{3}}{2} \) inches. So, the circumference is about \( 3\sqrt{3}\pi = 16.33 \) inches.

35. Arc length = \( \frac{\theta}{2\pi} \cdot 2\pi r \)

Arc length = \( \frac{3\pi}{4} \cdot 4 \)

Arc length = \( 3\pi \approx 9.4 \) in.

36. The circumference of circle \( P \) is \( CE \cdot \pi \).

The arc length of \( DE \) is \( \frac{1}{2} \cdot 2 \cdot CE \cdot \pi = CE \cdot \pi \).

Therefore, the circumference of circle \( P \) and the arc length of \( DE \) are equal because the radius of circle \( C \) is the same as the diameter of circle \( P \).

37. yes; Sample answer: The circumference of the red circle can be found using \( 2 = \frac{30°}{360°} \cdot C \). The circumference of the blue circle is double the circumference of the red circle.

b. Original: Arc length \( \overline{EF} = \frac{x}{360°} \cdot 2\pi 

Doubled arc measure: Arc length \( \overline{EF} = \frac{2x}{360°} \cdot 2\pi 

= 2 \left( \frac{1}{3} \cdot 2\pi \right) 

By factoring a 2, the arc length is 2 times the original arc length \( \overline{EF} \).

Therefore, when the arc measure is doubled, the arc length is doubled.

29. yes; Sample answer: The arc length also depends on the radius.

30. Let \( A \) = Alexandria.

Let \( S \) = Syene.

Let \( C \) = Earth’s Circumference.

From the proportional relationship \( \frac{575 \text{ mi}}{C} = \frac{7.2°}{360°} \), the circumference of Earth can be derived.

\[ 575 \cdot 360° = 7.2° \cdot C \]

\[ C = \frac{575 \cdot 360°}{7.2°} = 28,750 \]

The Earth’s circumference is 28,750 miles.

31. B: Since the central angles are equal, their intercepted arcs are also equal. The length of \( PQ \) is twice the length of \( RS \), so \( m\angle PCQ \) is twice \( m\angle RCS \). Therefore, the ratio of \( \angle PCQ \) to \( \angle RCS \) is 2 to 1.

32. \[
\text{Arc Length}_1 = \frac{\text{Arc Measure}_1 \cdot \text{Circumference}_1}{360°} \\
\text{Arc Length}_2 = \frac{\text{Arc Measure}_2 \cdot \text{Circumference}_2}{360°} \\
1 = \frac{\text{Arc Measure}_1 \cdot \text{Circumference}_1}{\text{Arc Measure}_2 \cdot \text{Circumference}_2} \\
1 = \frac{45° \cdot 2 \cdot \pi \cdot r_1}{30° \cdot 2 \cdot \pi \cdot r_2} \\
1 = \frac{3r_1}{2r_2} \\
\frac{2}{3} = \frac{r_1}{r_2}
\]

The ratio of the radius \( r_1 \) of circle \( C \) to the radius \( r_2 \) of circle \( P \) is \( \frac{2}{3} \).

33. One revolution of the larger gear is \( 2 \cdot 7 \cdot \pi = 14\pi \).

One revolution of the smaller gear is \( 2 \cdot 3 \cdot \pi = 6\pi \).

The smaller gear will complete \( 14\pi \div 6\pi = 2\frac{1}{3} \) revolutions during one single revolution of the larger gear.
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38. a. When the time is 1:30 P.M., the hour hand is between the one and two and the minute hand is on the six. The minute hand is 135° away from the hour hand or 4.5 hour mark.

\[
\frac{4.5}{12} \cdot 2\pi = \frac{9\pi}{12} = \frac{3\pi}{4}\pi
\]

So, the measure in radians when the time is 1:30 P.M. is \(\frac{3\pi}{4}\).

b. When the time is 3:15 P.M., the hour hand and the minute hand are very close to each other but not right on top of each other. The minute hand is on the 3 and the hour hand is \(\frac{1}{4}\) of the distance between 3 and 4, which is \(\frac{1}{4}\) of 30° or 7.5 degrees beyond the 3 mark.

\[
\frac{1}{4} \cdot \frac{1}{12} = \frac{1}{48} \cdot 12 = \frac{\pi}{24}
\]

So, the measure in radians when the time is 3:15 P.M. is \(\frac{\pi}{24}\).

39. Circumference of circle A: \(2 \cdot 3x \cdot \pi = 6\pi x\)
Circumference of circle B: \(2 \cdot x \cdot \pi = 2\pi x\)
Circumference of circle C: \(2 \cdot 5x \cdot \pi = 10\pi x\)
Total circumference of the three circles:

\[
6\pi x + 2\pi x + 10\pi x = 18\pi x
\]

\[
x = \frac{6\pi x}{18\pi} = \frac{7}{2} = 3.5
\]

\(AC = 3x + 5x = 8x = 8 \cdot 3.5 = 28\)

The distance between the centers of circle A and circle C is 28 units.

40. \(\pi\) is not a rational number because it cannot be written in the form \(\frac{a}{b}\), where \(a\) and \(b\) are real numbers.

\[
\frac{355}{113} = 3.14159292\ldots
\]

\(\pi = 3.141592654\ldots\)

The fraction \(\frac{355}{113}\) is the same as \(\pi\) to 6 decimal places.

Sample answer: The fraction \(\frac{104,348}{33,215}\) gives a slightly closer approximation to \(\pi\) to 9 decimal places.

41. Given \(\overline{FG} \equiv \overline{GH}, \angle JK = \angle KF\)

Prove \(JK\) and \(NG\) have the same length.

Sample answer:

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (FG \equiv GH), (\angle JK = \angle KF)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (FG = GH)</td>
<td>2. Definition of congruent segments</td>
</tr>
<tr>
<td>3. (FH = FG + GH)</td>
<td>3. Segment Addition Postulate (Post. 1.2)</td>
</tr>
<tr>
<td>4. (FH = 2FG)</td>
<td>4. Substitution Property of Equality</td>
</tr>
<tr>
<td>5. (m\angle JK = m\angle KF)</td>
<td>5. Definition of congruent angles</td>
</tr>
<tr>
<td>6. (m\angle JFL = m\angle JK + m\angle KFL)</td>
<td>6. Angle Addition Postulate (Post. 1.4)</td>
</tr>
<tr>
<td>7. (m\angle JFL = 2m\angle JK)</td>
<td>7. Substitution Property of Equality</td>
</tr>
<tr>
<td>8. (\angle NFG \equiv \angle JFL)</td>
<td>8. Vertical Angles Congruence Theorem (Thm. 2.6)</td>
</tr>
<tr>
<td>9. (m\angle NFG = m\angle JFL)</td>
<td>9. Definition of congruent angles</td>
</tr>
<tr>
<td>10. (m\angle NFG = 2m\angle JK)</td>
<td>10. Substitution Property of Equality</td>
</tr>
<tr>
<td>11. arc length of (\overline{JK})</td>
<td>11. Formula for arc length</td>
</tr>
<tr>
<td>(m\angle JK) (\frac{360^\circ}{360^\circ}) (2\pi FH)</td>
<td>arc length of (\overline{NJ}) (\frac{360^\circ}{360^\circ}) (2\pi FG)</td>
</tr>
<tr>
<td>12. arc length of (\overline{JK})</td>
<td>12. Substitution Property of Equality</td>
</tr>
<tr>
<td>(m\angle JK) (\frac{360^\circ}{360^\circ}) (2\pi(2FG))</td>
<td>arc length of (\overline{NG}) (\frac{360^\circ}{360^\circ}) (2\pi FG)</td>
</tr>
<tr>
<td>13. arc length of (\overline{NG})</td>
<td>13. Transitive Property of Equality</td>
</tr>
</tbody>
</table>

| arc length of \(\overline{NK}\) | arc length of \(\overline{JK}\) |
| 42. a. One arc length is \(\frac{180^\circ}{360^\circ}\) \(2\pi r = \pi r\). So, the sum of the four arc lengths is \(4\pi r\).

b. If \(\overline{AB}\) was divided into 8 congruent segments, the sum of the arc lengths is \(4\pi r\). If \(\overline{AB}\) was divided into 16 congruent segments, the sum of the arc lengths is still \(4\pi r\). For \(n\) congruent segments, the sum is still \(4\pi r\). The length of \(\overline{AB}\) stays the same no matter how many times you divide it into congruent segments. |
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43. The height is \(|4 - (-1)| = |5| = 5\) and the base is \(|8 - 2| = |6| = 6\). So, the area is \(\frac{1}{2} \cdot 5 \cdot 6 = 15\) square units.

44. The height is \(|1 - (-5)| = |1 + 5| = 6\) and the base is \(|-3 - 4| = |7| = 7\) units. Therefore, the area is \(6 \cdot 7 = 42\) square units.

11.2 Explorations (pp. 601–605)

1. Area = \(\pi r^2\)
   \[
   = \pi \cdot 6^2 = 36\pi \approx 113.10 \text{ units}^2
   \]

2. Area = \(\frac{1}{4}\pi r^2\)
   \[
   = \frac{1}{4}\pi \cdot 6^2 = \frac{36\pi}{4} = 9\pi \approx 28.27 \text{ units}^2
   \]

3. Area = \(\frac{7}{8}\pi r^2\)
   \[
   = \frac{7}{8}\pi \cdot 2^2 = \frac{7 \cdot 4\pi}{8} = \frac{7\pi}{2} = 11.00 \text{ units}^2
   \]

4. Area = \(\frac{2}{3}\pi r^2\)
   \[
   = \frac{2}{3}\pi \cdot 8^2 = \frac{2 \cdot 64\pi}{3} = \frac{128\pi}{3} \approx 134.04 \text{ units}^2
   \]

5. Area of sector = \(\frac{120^\circ}{360^\circ} \cdot 14^2 \cdot \pi \approx 205.25\)
   The area of the red sector is about 205.25 square feet.

6. Area of sector = \(\frac{360^\circ - 120^\circ}{360^\circ} \cdot 14^2 \cdot \pi \approx 410.50\)
   The area of the blue sector is about 410.5 square feet.

7. Area of sector \(\frac{m\angle FHG}{360^\circ} \cdot \text{Area of } \odot H\)
   \[
   = \frac{240}{360} \cdot \frac{85^\circ}{360} \cdot 214.37 = \frac{85}{360} \cdot 214.37 \cdot \frac{85^\circ}{360} \approx 907.92
   \]
   The area of \(\odot H\) is about 907.92 square centimeters.

8. The area of the semicircle is \(\frac{1}{2} \cdot \pi (3.5)^2 \approx 19.24\) square meters.
   The area of the triangle is \(\frac{1}{2} \cdot 7 \cdot 7 = 24.5\) square meters.
   Area of the semicircle + Area of the triangle
   \[
   \approx 19.24 + 24.5 = 43.74 \text{ square meters}
   \]
   The area of the figure is about 43.74 square meters.

9. yes; Solve the formula for the measure of the intercepted arc.
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11.2 Exercises (pp. 606–608)

**Vocabulary and Core Concept Check**

1. A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

2. Area of sector \(\frac{\text{Arc measure}}{360^\circ} \cdot \pi r^2\)

   Area of sector \(\frac{2 \cdot \text{Arc measure}}{360^\circ} \cdot \pi r^2\)

   \(= 2 \cdot \left(\frac{\text{Arc measure}}{360^\circ} \cdot \pi r^2\right)\)

   Multiplying the arc measure by 2 will double the area of the sector.

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3. \(A = \pi r^2\)
   \(= \pi (0.4)^2\)
   \(= 0.16\pi\)
   \(\approx 0.50\)

   The area of the circle is about 0.50 square centimeter.

4. radius \(= \frac{20}{2} = 10\)
   \(A = \pi r^2\)
   \(= \pi (10)^2\)
   \(= 100\pi\)
   \(\approx 314.16\)

   The area of the circle is about 314.16 square inches.

5. \(A = \pi r^2\)
   \(= \pi (5)^2\)
   \(= 25\pi\)
   \(\approx 78.54\)

   The area of the circle is about 78.54 square inches.

6. radius \(= \frac{16}{2} = 8\)
   \(A = \pi r^2\)
   \(= \pi (8)^2\)
   \(= 64\pi\)
   \(\approx 201.06\)

   The area of the circle is about 201.06 square feet.

7. \(A = \pi r^2\)
   \(89 = \pi r^2\)
   \(\frac{89}{\pi} = r^2\)
   \(5.32 = r\)

   The circle has a radius of about 5.32 feet.

8. \(A = \pi r^2\)
   \(380 = \pi r^2\)
   \(\frac{380}{\pi} = r^2\)
   \(11.00 = r\)

   The circle has a radius of about 11 inches.

9. \(A = \pi r^2\)
   \(12.6 = \pi r^2\)
   \(\frac{12.6}{\pi} = r^2\)
   \(2.00 = r\)

   The circle has a diameter of about 4 inches.

10. \(A = \pi r^2\)
    \(676\pi = \pi r^2\)
    \(\frac{676\pi}{\pi} = r^2\)
    \(26 = r\)

    The circle has a diameter of 52 centimeters.

11. The area of the region is \(\pi (12)^2 = 144\pi = 452.39\) square miles.

    Population density \(= \frac{\text{Number of people}}{\text{Area of land}}\)
    \(= \frac{210,000}{452.39}\)
    \(\approx 464.20\)

    The population density is about 464 people per square mile.

12. The area of the region is \(\pi (6)^2 = 36\pi \approx 113.1\) square miles.

    The population density is:
    \(\frac{650,000}{113.1}\)
    \(\approx 5747.1\)

    The population density is about 5747 people per square mile.

13. The area of the region is \(\pi (4)^2 = 16\pi \approx 50.27\) square miles.

    \(50.27 \cdot 6366 = 319,990\)

    The number of people who live in the region is about 319,990.

14. \(\frac{513}{\text{Area of land}} = \frac{79,000}{513\pi}\)

    \(r = \sqrt{\frac{79,000}{513\pi}} \approx 7\)

    The radius of the circular region is about 7 miles.

15. Area of sector \(= \frac{60^\circ}{360^\circ} \cdot 10^2 \cdot \pi \approx 52.36\)

    The area of the red sector is about 52.36 square inches.

    Area of sector \(= \frac{360^\circ - 60^\circ}{360^\circ} \cdot 10^2 \cdot \pi\)
    \(= \frac{300}{360} \cdot 100 \cdot \pi\)
    \(\approx 261.80\)

    The area of the blue sector is about 261.8 square inches.
16. Area of sector \(= \frac{360° - 256°}{360°} \cdot 14^2 \cdot \pi \)
\[= \frac{104}{360} \cdot 196 \cdot \pi \]
\[= 177.88 \]
The area of the red sector is about 177.88 square centimeters.

Area of sector \(= \frac{256°}{360°} \cdot 14^2 \cdot \pi \)
\[= \frac{224}{360} \cdot 784 \cdot \pi \]
\[= 437.87 \]
The area of the blue sector is about 437.87 square centimeters.

17. Area of sector \(= \frac{137°}{360°} \cdot 28^2 \cdot \pi \)
\[= \frac{235}{360} \cdot 784 \cdot \pi \]
\[= 937.31 \]
The area of the red sector is about 937.31 square meters.

Area of sector \(= \frac{360° - 137°}{360°} \cdot 28^2 \cdot \pi \)
\[= \frac{223}{360} \cdot 784 \cdot \pi \]
\[= 1525.70 \]
The area of the blue sector is about 1525.7 square meters.

18. Area of sector \(= \frac{75°}{360°} \cdot 4^2 \cdot \pi \)
\[= \frac{25}{360} \cdot 16 \cdot \pi \]
\[= 39.79 \]
The area of the blue sector is about 39.79 square feet.

19. The diameter was substituted in the formula for area instead of the radius. The diameter is 12 feet, so the radius is 6 feet.
\[A = \pi r^2 \]
\[= \pi (6)^2 \]
\[= 113.10 \text{ ft}^2 \]

20. The angle measures should be on the same side of the proportion.
\[
\frac{115}{360} = \frac{n}{255} \\
81.46 \text{ ft}^2 = n
\]

21. Area of sector \(JLK = \frac{m\angle JKL}{360°} \cdot \text{Area of } \odot M \)
\[56.87 = \frac{360° - 50°}{360°} \cdot \text{Area of } \odot M \]
\[56.87 = \frac{310°}{360°} \cdot \text{Area of } \odot M \]
\[56.87 \cdot \frac{360°}{310°} = \text{Area of } \odot M \]

Area of \(\odot M \approx 66.04 \)
The area of \(\odot M \) is about 66.04 square centimeters.

22. Area of sector \(JMK = \frac{m\angle JMK}{360°} \cdot \text{Area of } \odot M \)
\[
12.36 = \frac{89°}{360°} \cdot \pi r^2 \\
\frac{360° \cdot 12.36}{89°} = \pi r^2 \\
\frac{50}{\pi} = r^2 \\
r \approx 3.99
\]
The radius of \(\odot M \) is about 3.99 meters.

23. Area of the larger circle:
\[A = \pi r^2 \]
\[= \pi (24)^2 \]
\[= 576\pi \]

Area of the smaller circle:
\[A = \pi r^2 \]
\[= \pi (6)^2 \]
\[= 36\pi \]

Area of the larger circle – Area of the smaller circle
\[= 576\pi - 36\pi \]
\[= 540\pi \]
\[= 1696.46 \]

The area of the shaded region is about 1696.46 square meters.

24. The radius of each circle is \(\frac{20}{4} = 5 \).

Area of one circle:
\[A = \pi r^2 \]
\[= \pi (5)^2 \]
\[= 25\pi \]

The sum of the areas of the four circles \(4 \cdot 25\pi = 100\pi \).

Area of the square:
\[A = s^2 \]
\[= (20)^2 \]
\[= 400 \]

Area of the square – Area of the 4 circles
\[= 400 - 100\pi \]
\[= 85.84 \]

The area of the shaded region is about 85.84 square inches.
Chapter 11

25. The radius of the larger circle is 5 feet.
   \begin{align*}
   \text{Area of the fifth circle:} & \quad A = \pi r^2 \\
   & = \pi (5)^2 \\
   & = 25\pi
   \\
   \text{Area of the fourth circle:} & \quad A = \pi r^2 \\
   & = \pi (4)^2 \\
   & = 16\pi
   \\
   \text{The area of the largest blue region is } & \quad 25\pi - 16\pi = 9\pi \text{ square feet.}
   \\
   \text{Area of the third circle:} & \quad A = \pi r^2 \\
   & = \pi (3)^2 \\
   & = 9\pi
   \\
   \text{Area of the second circle:} & \quad A = \pi r^2 \\
   & = \pi (2)^2 \\
   & = 4\pi
   \\
   \text{The area of the blue region between the second and third circles is } & \quad 9\pi - 4\pi = 5\pi \text{ square feet.}
   \\
   \text{The sum of the two blue regions is } & \quad 9\pi + 5\pi = 14\pi = 43.98.
   \\
   \text{The area of the shaded region is about } & \quad 43.98 \text{ square feet.}
   \\
   \end{align*}

26. Area of the larger semicircle:
   \begin{align*}
   \text{Area of sector} & = \frac{\text{Arc measure}}{360^\circ} \times \text{Area of circle} \\
   & = \frac{180^\circ}{360^\circ} \times \pi \times 16^2 \\
   & = 128\pi
   \\
   \text{Area of the smaller semicircle:} & \quad \text{Area of sector} = \frac{\text{Arc measure}}{360^\circ} \times \text{Area of circle} \\
   & = \frac{180^\circ}{360^\circ} \times \pi \times 8^2 \\
   & = 32\pi
   \\
   \text{Area of the larger semicircle} - \text{Area of the smaller circle} & \quad = 128\pi - 32\pi \\
   & = 96\pi
   \\
   \text{The area of the shaded region is about } & \quad 301.59 \text{ square centimeters.}
   \\
   \end{align*}

27. Radius of the circle:
   \begin{align*}
   2x^2 & = 5^2 \\
   x & = \sqrt{\frac{25}{2}} \\
   & = \frac{5\sqrt{2}}{2}
   \\
   \text{Area of the triangle:} & \quad A = \frac{1}{2} \times \frac{5\sqrt{2}}{2} \times \left(2 \times \frac{5\sqrt{2}}{2}\right) \\
   & = \frac{25\sqrt{2} \times 2}{4} = \frac{50}{4} = 12.5
   \\
   \text{Area of the circle:} & \quad A = \pi r^2 \\
   & = \pi \left(\frac{5\sqrt{2}}{2}\right)^2 \\
   & = \pi \cdot \frac{25}{2} = 12.5\pi
   \\
   \text{Area of the circle} - \text{Area of the triangle} & \quad = 12.5\pi - 12 = 7.63
   \\
   \text{The area of the shaded region is about } & \quad 7.63 \text{ square meters.}
   \\
   \end{align*}

28. The length of the sides of the triangles are 3, 4, and \(\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5\).
   \begin{align*}
   \text{The area of one of the triangles is } & \quad \frac{1}{2} \times 3 \times 4 = 6 \text{ square meters.}
   \\
   \text{The sum of the areas of the two triangles is } & \quad 2 \times 6 = 12 \text{ square meters.}
   \\
   \text{The area of the circle is } & \quad \pi (2.5)^2 = 6.25\pi \text{ square meters.}
   \\
   \text{Area of the circle} - \text{Areas of the two triangles} & \quad = 6.25\pi - 12 = 7.63
   \\
   \text{The area of the shaded region is about } & \quad 7.63 \text{ square meters.}
   \\
   \end{align*}

29. The area of the square is \((3x - 2)^2\) square feet.
   \begin{align*}
   \text{The area of the quarter circle is } & \quad \frac{1}{4}(3x - 2)^2\pi \text{ square feet.}
   \\
   \text{The area of the rectangle is } & \quad 5x \times (2x + 1) \text{ square feet.}
   \\
   3x - 2 & = 2x + 1 \\
   x & = 3
   \\
   3x - 2 & = 3 \times 3 - 2 = 7
   \\
   2x + 1 & = 2 \times 3 + 1 = 7
   \\
   5x & = 5 \times 3 = 15
   \\
   \text{Area of the square} + \text{Area of the quarter circle} & \quad + \text{Area of the rectangle} \\
   & = 7^2 + \frac{1}{4} \times \pi \times 7^2 + 15 \times 7 \\
   & = 154 + 12.25 \times \pi \\
   & = 192.48
   \\
   \text{The total area of the shape of a putting green at a miniature golf course is about } & \quad 192.48 \text{ square feet.}
   \\
   \end{align*}
30. no; If the radius is doubled, the area is multiplied by 4.
   \[ A = \pi (2r)^2 = \pi \cdot 4 \cdot r^2 = 4 \cdot \pi r^2 \]

31. a. Area of sector \( = \frac{145^\circ}{360^\circ} \cdot 15^\circ \cdot \pi \)
   \[ \approx 284.71 \]
   The area of the lawn that is covered by the sprinkler is about 284.71 square feet.

b. Area of sector \( = \frac{145^\circ}{360^\circ} \cdot 12^\circ \cdot \pi \)
   \[ \approx 182.21 \]
   The area of the lawn that is covered by the sprinkler is about 182.21 square feet.

32. a. Area of sector \( = \frac{245^\circ}{360^\circ} \cdot 18^\circ \cdot \pi \)
   \[ \approx 692.72 \]
   The area of the water that is covered by the lighthouse is about 692.72 square miles.

b. Area of sector \( = \frac{360^\circ - 245^\circ}{360^\circ} \cdot 18^\circ \cdot \pi \)
   \[ \approx 325.5 \]
   The area of the land that is covered by the lighthouse is about 325.5 square miles.

33. \( C_1 : C_2 = \pi r_1 : \pi r_2 \)
   \( C_1 = r_1 \)
   \( C_2 = r_2 \)

Circles are similar if the ratio of their circumferences are equal to the ratios of their radii. Circles circumferences are proportional to their radii. All circles are similar.

\[ \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} \]
\[ = \frac{r_1^2}{r_2^2} \]

Two circles are similar if the ratio of their areas is equal to the ratio of the square of their radii. Circles are proportional to the square of their radii.

34. The radius of the larger circle is \( \sqrt{2} \cdot \text{side length} \).
   Area of the larger circle:
   \[ A_1 = \pi \left(\frac{\sqrt{2}}{2}\right)^2 = \pi \cdot \frac{1}{2} = \frac{\pi}{2} \]
   The radius of the smaller circle is \( \frac{1}{2} \cdot \text{side length} \).
   Area of the smaller circle:
   \[ A_2 = \pi \left(\frac{1}{2}\right)^2 = \pi \cdot \frac{1}{4} = \frac{\pi}{4} \]
   \[ \frac{A_1}{A_2} = \frac{\frac{\pi}{2}}{\frac{\pi}{4}} = 2 \]

The ratio of the area of the larger circle to the area of the smaller circle is 2:1.

35. a. A circle graph is appropriate because the data you are comparing are percentages of that total 100%.

b. The central angle for the sector representing bussing measures \( 0.65 \cdot 360^\circ = 234^\circ \).

The central angle for the sector representing walking measures \( 0.25 \cdot 360^\circ = 90^\circ \).

The central angle for the sector representing other measures \( 0.10 \cdot 360^\circ = 36^\circ \).

36. yes; Sample answer: The side length of each of the small squares is \( \frac{1}{8} \) the side length of the outer square, and each small square is either completely colored or has a quarter circle colored.
Chapter 11

37. If a 12-inch pizza is divided among 3 people equally, each person receives \( \frac{3 \pi r}{3} \) or about 37.7 square inches.

8 people will require a minimum of about

8 \( \times \) 37.7 = 301.6 square inches.

A 10-inch pizza is \( \pi (5)^2 = 78.54 \) square inches and costs about 6.99 \( \pi / 78.54 \approx 8.9 \) cents per square inch.

A 14-inch pizza is \( \pi (7)^2 = 153.94 \) square inches and costs about 12.99 \( \pi / 153.94 \approx 8.4 \) cents per square inch.

a. Two 14-inch pizzas cost 2 \( \times \) 12.99 = $25.98 and the area of both, 2 \( \times \) 153.94 = 307.88 square inches, will be enough pizza for 8 people.

b. Two 10-inch pizzas and one 14-inch pizza cost

2 \( \times \) 6.99 + 10.99 = $26.97 and allows for three toppings.

The total area is 2 \( \times \) 78.54 + 153.94 = 311.02 square inches, which is enough for 8 people. Three 10-inch pizzas cost 8 \( \times \) 6.99 = $20.97, but has an area of only 3 \( \times \) 78.54 = 235.62 square inches.

c. Four 10-inch pizzas cost 4 \( \times \) 6.99 = $27.96 and because the circumference to area ratio is greater for small circles, the outer crust will be maximized.

38. The area of an ellipse with major axis \( a \) and minor axis \( b \) is \( \pi \cdot a \cdot b \). If \( a = b \), then the ellipse is a circle and the area is \( \pi a^2 \) or \( \pi b^2 \).

39. a.

<table>
<thead>
<tr>
<th>( x )</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6.54 in.²</td>
<td>13.09 in.²</td>
<td>19.63 in.²</td>
<td>26.18 in.²</td>
<td>32.72 in.²</td>
<td>39.27 in.²</td>
</tr>
</tbody>
</table>

b. The measures of \( \angle A, \angle B, \) and \( \angle C \) are then 60°. So the area of each sector of the three circles is \( \frac{60^\circ}{360^\circ} \times 36\pi = 6\pi \) square inches and total area of the three sectors is 3 \( \times \) 6\( \pi = 18\pi \) square inches.

Area of triangle – Areas of three sectors

\( 36\sqrt{3} - 18\pi \approx 5.81 \)

The area of the shaded region between the three tangent circles is about 5.81 square inches.

d. Circle with radius 5 in.:

<table>
<thead>
<tr>
<th>( x )</th>
<th>30°</th>
<th>60°</th>
<th>90°</th>
<th>120°</th>
<th>150°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4.71 in.²</td>
<td>7.07 in.²</td>
<td>9.42 in.²</td>
<td>11.78 in.²</td>
<td>14.14 in.²</td>
<td></td>
</tr>
</tbody>
</table>

3. \( 4.71 - 2.36 = 0.078 \)

60° – 30°

7.07 – 4.71 = 0.079

90° – 60°

9.42 – 7.07 = 0.078

120° – 90°

11.78 – 9.42 = 0.079

150° – 120°

14.14 – 11.78 = 0.079

180° – 150°

The relationship between \( x \) and \( y \) is linear because the rate of change is constant.

40. Since the radius of each circle is 6, each side of the triangle is 12, and therefore, the triangle is equilateral. The area of one circle is \( \pi (6)^2 = 36\pi \) square inches.

The height of the triangle is \( \sqrt{12^2 - 6^2} = \sqrt{108} = 6\sqrt{3} \) inches and the base is 12. Therefore, the area is \( \frac{1}{2} \times 12 \times 6\sqrt{3} = 36\sqrt{3} \) square inches.

The measures of \( \angle A, \angle B, \) and \( \angle C \) are then 60°. So the area of each sector of the three circles is \( \frac{60^\circ}{360^\circ} \times 36\pi = 6\pi \) square inches and total area of the three sectors is 3 \( \times \) 6\( \pi = 18\pi \) square inches.

Area of triangle – Areas of three sectors

\( 36\sqrt{3} - 18\pi \approx 5.81 \)

Yes, the areas changed. No, the sector area to arc measure relationship holds for any radius.

41. Sample answer: Let 2\( a \) and 2\( b \) represent the lengths of the legs of the triangle. The areas of the semicircles are \( \frac{1}{2} \pi a^2 \), \( \frac{1}{2} \pi b^2 \), and \( \frac{1}{2} \pi (a^2 + b^2) \). The area of the crescents is the area of the shaded regions from both sides leaves the area of the crescents on the left and the area of the triangle on the right.

Maintaining Mathematical Proficiency

42. \( A = \frac{1}{2} bh \)

\( = \frac{1}{2} \times 18 \times 6 \)

\( = 54 \)

The area of the triangle is 54 square inches.
43. \( A = \frac{1}{2}h(b_1 + b_2) \)
\[ = \frac{1}{2} \cdot 7(4 + 10) \]
\[ = \frac{1}{2} \cdot 7(14) \]
\[ = 49 \]
The area of the trapezoid is 49 square feet.

44. \( A = \frac{1}{2}bh \)
\[ = \frac{1}{2} \cdot 9 \cdot 13 \]
\[ = 58.5 \]
The area of the triangle is 58.5 square inches.

45. \( A = bh \)
\[ = 5(3) \]
\[ = 15 \]
The area of the parallelogram is 15 square feet.

11.3 Explorations (p. 609)
1. a. This equilateral triangle has side lengths of 4 and angle measures of 60°.

Use perpendicular bisectors to find the center. Using the 30°-60°-90° Triangle Theorem (Thm. 9.5), the apothem is opposite the 60° angle, and the side opposite the 30° angle is 2.

\[ \text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \]
\[ \text{shorter leg} = \frac{\text{longer leg}}{\sqrt{3}} \]

The apothem is \(\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}\). So, the total area of the triangle is \(\frac{1}{2} \cdot \frac{2\sqrt{3}}{3} \cdot 4 \cdot 3 = 4\sqrt{3} \approx 6.93\) square units.

b. Each vertex angle of a regular pentagon measures 108°. When each angle is bisected, the angle measures of the resulting triangles are 90°, 36°, and 54°. To find the apothem, use the tangent function.

\[ \tan 54° = \frac{a}{2} \]
\[ a = 2 \cdot \tan 54° \approx 2.75 \]
The area of one triangle is \(\frac{1}{2} \cdot 2.75 \cdot 4 = 5.5\). The total area of the pentagon is \(5 \cdot 5.5 = 27.5\) square units.

c. Each vertex angle of a regular hexagon measures 120°. When each angle is bisected, the angle measures of the resulting triangles are 90°, 30°, and 60°. Using the 30°-60°-90° Triangle Theorem (Thm. 9.5), the apothem is opposite the 60° angle, and the side opposite the 30° angle is 2.

\[ \text{longer leg} = \text{shorter leg} \cdot \sqrt{3} \]
\[ \text{shorter leg} = \frac{\text{longer leg}}{\sqrt{3}} \]

The apothem is \(2 \cdot \sqrt{3}\).
The area of one triangle is \(\frac{1}{2} \cdot 2\sqrt{3} \cdot 4 = 4\sqrt{3}\).
The total area of the hexagon is \(6 \cdot 4\sqrt{3} = 24\sqrt{3} \approx 41.57\) square units.
11.3 Monitoring Progress (pp. 610–613)

d. Each vertex angle of a regular octagon measures 135°. When each angle is bisected, the angle measures of the resulting triangles are 90°, 67.5°, and 22.5°. Use the tangent function to find the apothem.

\[
\tan 67.5° = \frac{a}{2} \\
2 \cdot \tan 67.5° = a \\
a \approx 4.83
\]

The apothem is about 4.83 units and the area of one triangle is \(\left(\frac{1}{2} \cdot 4.83 \cdot 4\right) = 9.66\) square units. The total area of the octagon is \(8 \cdot 9.66 = 77.28\) square units.

2. Determine the center of the polygon. Find the distance from the center to a side of the polygon or the apothem. Find the area of one triangle and multiply that area by the number of sides of the polygon.

3. Multiply the apothem by the perimeter of the polygon and divide by 2.

4. Area = \(\frac{1}{2} \cdot 4.13 \cdot 6 \cdot 5 = 61.95\)

The area of the pentagon is 61.95 square meters.

5. Each vertex angle measures \(\frac{3 \cdot 180°}{5} = 108°\).

The radius of the polygon bisects the vertex angle into two equal angles, measuring 54° each.

Use the Pythagorean Theorem to find the base of the triangle.

\[a^2 + b^2 = c^2\]

6.5² + \(b^2 = 8^2\)

\[b^2 = 64 - 42.25\]

\[b^2 = 21.75\]

\[b = 4.66\]

Therefore, the side length of the pentagon is about 2 \(\cdot 4.66 = 9.32\) units.

\[A = \frac{1}{2}aP\]

\[= \frac{1}{2}(6.5)(5 \cdot 9.32)\]

\[= 151.45\]

The area of the regular pentagon is about 151.45 square units.

6. The central angle measures \(\frac{360°}{10} = 36°\). Since the triangle formed by the radii of the polygon is isosceles, each base angle measures 72°. The base (side of the polygon) is divided into two equal parts, 3.5. The apothem can be found using \(\tan 72° = \frac{a}{d}\). So, \(a = 3.5 \cdot \tan 72° = 10.77\).

\[A = \frac{1}{2}aP\]

\[= \frac{1}{2}(10.77)(7 \cdot 10)\]

\[= 376.95\]

The area of the dodecagon is about 376.95 square units.

11.3 Exercises (pp. 614–616)

Vocabulary and Core Concept Check

1. Divide 360° by the number of sides of the polygon in order to find the central angle measure.

2. The statement that is different is “Find the apothem of polygon ABCDE.”

The length of apothem \(FG\) is 5.5 units.

The radius of the polygon \(ABCD\) is represented by \(EF = AF = 6.8\) units.

Monitoring Progress and Modeling with Mathematics

3. \(A = \frac{1}{2}d_1d_2\)

\[= \frac{1}{2} \cdot 38 \cdot 19\]

\[= 361\]

The area of the kite is 361 square units.

4. \(A = \frac{1}{2}d_1d_2\)

\[= \frac{1}{2} \cdot 12 \cdot 12\]

\[= 72\]

The area of the kite is 72 square units.
5. \( A = \frac{1}{2} d_1 d_2 \)
   \[ = \frac{1}{2} \cdot 10 \cdot 14 \]
   \[ = 70 \]
The area of the rhombus is 70 square units.

6. \( d_1 = 6, d_2 = 8 \) (using a 3-4-5 triangle)
   \[ A = \frac{1}{2} d_1 d_2 \]
   \[ = \frac{1}{2} \cdot 6 \cdot 8 \]
   \[ = 24 \]
The area of the rhombus is 24 square units.

7. The center of polygon \( JKLMN \) is \( P \).

8. \( \angle NPM \) is a central angle.

9. \( PN \) and \( PM \) are radii of length 5 of circle \( P \).

10. The length of apothem \( PQ \) is 4.05 units.

11. The measure of a central angle of a polygon with 10 sides
    is \( \frac{360^\circ}{10} = 36^\circ \).

12. The measure of a central angle of a polygon with 18 sides
    is \( \frac{360^\circ}{18} = 20^\circ \).

13. The measure of a central angle of a polygon with 24 sides
    is \( \frac{360^\circ}{24} = 15^\circ \).

14. The measure of a central angle of a polygon with 7 sides
    is \( \frac{360^\circ}{7} = 51.4^\circ \).

15. The measure of a central angle of a regular octagon is
    \( \frac{360^\circ}{8} = 45^\circ \), so \( m\angle GJH = 45^\circ \).

16. \( m\angle GJK = \frac{45^\circ}{2} = 22.5^\circ \)

17. \( m\angle KJG = \frac{180^\circ - 45^\circ}{2} = 67.5^\circ \)

18. \( m\angle EJH = 45^\circ \cdot 3 = 135^\circ \)

19. \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 2\sqrt{3} \cdot 3 \cdot 12 \]
    \[ = 36\sqrt{3} \approx 62.35 \]
The area of the triangle is about 62.35 square units.

20. The apothem is \( a = \sqrt{10^2 - 3.42^2} \approx 9.4 \).
    \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 9.4 \cdot 9 \cdot 6.84 \]
    \[ = 289.33 \]
The area of the nonagon is about 289.33 square units.

21. Find the side length of the heptagon.
    \[ a = \sqrt{2.77^2 - 2.5^2} \approx 1.19 \]
    So, the side length is \( 2 \cdot 1.19 = 2.38 \).
    \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 2.5 \cdot 7 \cdot 2.38 \]
    \[ = 20.83 \]
The area of the heptagon is about 20.83 square units.

22. The central angle of the hexagon is \( \frac{360^\circ}{6} = 60^\circ \).
    The apothem (height of the equilateral triangle) is \( 3.5\sqrt{3} \).
    \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 3.5\sqrt{3} \cdot 6 \cdot 7 \]
    \[ = 147 \sqrt{3} \approx 127.31 \]
The area of the hexagon is about 127.31 square units.

23. The central angle is \( \frac{360^\circ}{8} = 45^\circ \). Since the triangle formed
    by the radii of the polygon is isosceles, each base angle
    measures 67.5\(^\circ\). Find the apothem.
    \[ \sin 67.5^\circ = \frac{a}{11} \]
    \[ a = 11 \cdot \sin 67.5^\circ \approx 10.16 \]

    Find the length of the side of the octagon.
    \[ \cos 67.5^\circ = \frac{x}{11} \]
    \[ x \approx 4.2 \]

    Therefore, the side length is \( 2 \cdot 4.2 = 8.4 \) units.
    \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 10.16 \cdot 8 \cdot 8.4 \]
    \[ = 341.38 \]
The area of the octagon is about 341.38 square units.

24. The central angle is \( \frac{360^\circ}{5} = 72^\circ \). Since the triangle formed
    by the radii of the polygon is isosceles, each base angle
    measures 54\(^\circ\). The base (side of the polygon) is divided into
two equal parts. The apothem equals 5, so half of the side is
    \[ \tan 54^\circ = \frac{x}{5} \]
    \[ x = \frac{5}{\tan 54^\circ} \approx 3.6 \]  

The length of a side is \( 3.6 \cdot 2 = 7.2 \).
    \[ A = \frac{1}{2} a \cdot ns \]
    \[ = \frac{1}{2} \cdot 5 \cdot 5 \cdot 7.2 \]
    \[ = 90 \]
The area of the pentagon is about 90 square units.
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25. The side lengths were used instead of the diagonals.
\[ d_1 = 3 + 5 = 8, \quad d_2 = 2 + 2 = 4 \]
\[ A = \frac{1}{2} d_1 d_2 \]
\[ = \frac{1}{2} \cdot 8 \cdot 4 \]
\[ = 16 \]

So, the area of the kite is 16 square units.

26. Because 13 is the apothem, 7.5 is half the side length, not the entire side length.
\[ S = 2\sqrt{15^2 - 13^2} \]
\[ = 2\sqrt{56} \approx 15 \]
\[ A = \frac{1}{2} a \cdot ns \]
\[ \approx \frac{1}{2} \cdot 13 \cdot 6 \cdot 15 \]
\[ = 585 \]

The area of the hexagon is about 585 square units.

27. The central angle is 72°. Each base angle of the isosceles triangle measures 54°. Find the apothem.
\[ \tan 54° = \frac{a}{6} \]
\[ a = 6 \tan 54° \]
\[ a \approx 8.3 \]
\[ A = \frac{1}{2} a \cdot ns \]
\[ \approx \frac{1}{2} \cdot 8.3 \cdot 5 \cdot 12 \]
\[ = 249 \]

The circle has a radius of about \( r = \sqrt{8.3^2 + 6^2} \approx 10.2 \) units. So, the area of the circle is \( \pi \cdot 10.2^2 \approx 326.85 \) square units.

Area of the circle = Area of the pentagon
\[ \approx 326.85 - 249 = 77.85 \]

The area of the shaded region is about 77.85 square units.

28. The quadrilateral is a square and the diagonals are equal, so the area formula \( \frac{1}{2} d_1 d_2 \) can be used.
\[ A = \frac{1}{2} \cdot 28 \cdot 28 = 392 \]

The area of the circle is \( \pi \cdot 14^2 = 615.75 \) square units.

Area of the circle = Area of the square
\[ \approx 615.75 - 392 = 223.75 \]

The area of the shaded region is about 223.75 square units.

29. The smaller triangle is a 30°-60°-90° triangle. The longer leg of this triangle is \( 4\sqrt{3} \) and the side of the equilateral triangle is \( 2 \cdot 4\sqrt{3} = 8\sqrt{3} \). The shorter leg (apothem) is 4. The area of the equilateral triangle is
\[ A = \frac{1}{2} a \cdot ns = \frac{1}{2} \cdot 4 \cdot 3 \cdot 8\sqrt{3} = 48\sqrt{3} \]

The area of the circle is \( \pi \cdot 8^2 = 64\pi \).

Area of the circle = Area of the equilateral triangle
\[ = 64\pi - 8\sqrt{3} - 40\sqrt{3} = 117.92 \]

The area of the shaded region is about 117.92 square units.

30. The central angle is 60°, the apothem is the longer leg of the right triangle, \( 2\sqrt{3} \), and the hypotenuse (radius of the circle) is 4. The area of the sector is \( \frac{60°}{360°} \cdot \pi \cdot 4^2 = 8.38 \). The area of the triangle is \( \frac{1}{2} \cdot 2\sqrt{3} \cdot 4 = 4\sqrt{3} \).

Area of the sector = Area of the triangle
\[ \approx 8.38 - 4\sqrt{3} = 1.45 \]

The area of the shaded region is about 1.45 square units.

31. \( \tan 60° = \frac{a}{4} \]
\[ a = 4 \tan 60° \approx 6.93 \]
\[ A = \frac{1}{2} a \cdot ns \]
\[ \approx \frac{1}{2} \cdot 6.93 \cdot 6 \cdot 8 \]
\[ = 166.28 \]

The area of the hexagonal shape is about 166 square inches.

32. The central angle of a regular octagon is 45°. The base angles of the isosceles triangle formed by the radii of the polygon measure 67.5°. The apothem is 1.2 centimeters.
\[ \tan 67.5° = \frac{1}{2} \]
\[ x = \frac{1}{2} \cdot \tan 67.5° \approx 0.497 \]

The side has length \( 2 \cdot 0.497 = 0.994 \) centimeter.
\[ A = \frac{1}{2} a \cdot ns \]
\[ \approx \frac{1}{2} \cdot 1.2 \cdot 8 \cdot 0.994 \]
\[ = 4.77 \]

The octagonal background has an area of about 4.77 square centimeters. The circular face is 3.14 square centimeters. So, the border has an area of about 4.77 – 3.14 = 1.63 square centimeters.

33. true; Sample answer: As the number of sides increases, the polygon fills more of the circle.

34. true; Sample answer: The radius is the hypotenuse of each triangle.

35. false; Sample answer: The radius can be less than or greater than the side length.
36. Sample answer: B; A; B appears to have the largest area and A appears to have the smallest area.

Figure A: \[ A = \pi r^2 \]
\[ = \pi \cdot (6.5)^2 \]
\[ = 132.73 \text{ in}^2. \]

Figure B: The central angle of the pentagon is 72°. Since the triangle formed by the radii of the pentagon is isosceles, each base angle measures 54°. The apothem is \( a = 4.5 \tan 54° \approx 6.19 \). So, the area of the pentagon is about \( \frac{1}{2} \cdot 6.19 \cdot 5 \cdot 9 = 139.36 \) square inches.

Figure C: \[ A = \frac{1}{2} bh \]
\[ = \frac{1}{2}(18)(15) \]
\[ = 135 \text{ in}^2. \]

So, B has the greatest area and A has the least area.

37. Graph the circle and inscribe a regular pentagon in the circle.

![Graph of a circle with a regular pentagon inscribed within it]

38. The area of a kite is \( A = \frac{1}{2} d_1 d_2 \). If \( d_2 \) is doubled, then
\[ A = \frac{1}{2} d_1(2d_2) = 2 \left( \frac{1}{2} d_1 d_2 \right) \]
which is 2 times the original area. Therefore, when one of the diagonals is doubled, the area is doubled. When both diagonals are doubled, then
\[ A = \frac{1}{2} \cdot (2 \cdot d_1) \cdot (2 \cdot d_2) = 4 \left( \frac{1}{2} d_1 d_2 \right) \]
which is 4 times the original area. Therefore, when both diagonals are doubled, the area is quadrupled.

39. \[ A = \frac{1}{2} d_1 d_2 \]
\[ 324 = \frac{1}{2} d_1(2d_1) \]
\[ 324 = d_1^2 \]
\[ d_1 = \sqrt{324} = 18 \]

The two diagonals of the kite have lengths 18 inches and \( 2 \cdot 18 = 36 \) inches.

40. \[ A = \frac{1}{2} d_1 d_2 \]
\[ 98 = \frac{1}{2} d_1(4d_1) \]
\[ 98 = 2d_1^2 \]
\[ d_1 = \sqrt{49} = 7 \]

The two diagonals of the rhombus have lengths 7 feet and \( 4 \cdot 7 = 28 \) feet.

41. There is enough information to find the area. Since the 9-gon has a perimeter of 18 inches, each side is \( 18 \div 9 = 2 \) inches. The central angle of a 9-gon is 40°, when bisected is 20°. The apothem can be found by using the tangent function.
\[ \tan 20° = \frac{a}{l} \]
\[ a = -\frac{l}{\tan 20°} \approx 2.75 \]

Therefore, the area is about \( \frac{1}{2} \cdot 2.75 \cdot 9 \cdot 2 = 24.75 \) square inches.

42. no; Sample answer: A rhombus is not a regular polygon.

43. Given \( d_1 \) and \( d_2 \) are diagonals of quadrilateral \( PQRS \) and \( d_1 \perp d_2 \).

Prove Area \( = \frac{1}{2} d_1 d_2 \)

Area of \( \triangle PQR = \frac{1}{2} \cdot QT \cdot PR \)

Area of \( \triangle PSR = \frac{1}{2} \cdot ST \cdot PR \)

Area of \( PQRS = \) Area of \( \triangle PQR + \) Area of \( \triangle PSR \)
\[ = \frac{1}{2} \cdot QT \cdot PR + \frac{1}{2} \cdot ST \cdot PR \]
\[ = \frac{1}{2} \cdot PR(QT + ST) \]
\[ d_1 = QT + ST \]
\[ d_2 = PR \]

Area of \( PQRS = \frac{1}{2} \cdot d_2 \cdot d_1 \)

Therefore, the area is \( A = \frac{1}{2} d_1 d_2 \).
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44. Sample answer: Determine the area of each equilateral triangle by using a 30°-60°-90° triangle to determine the length of an altitude of one triangle. Then multiply the area of the triangle by 6.

45. In a square, the diagonals are equal and the length of each is $s \cdot \sqrt{2}$. The area of a rhombus can be found using the diagonals, $\frac{1}{2} \cdot d_1d_2$. Both diagonals are equal to $s \cdot \sqrt{2}$, therefore the area is $\frac{1}{2} \cdot (s\sqrt{2})(s\sqrt{2}) = \frac{1}{2} \cdot s^2 \cdot 2 = s^2$.

46. The altitude of an equilateral triangle is opposite 60°, therefore its measure is $\frac{s}{2} \cdot \sqrt{3}$. The area of the triangle is $\frac{1}{2} \cdot \frac{s}{2} \cdot \sqrt{3} \cdot s = \frac{s^2 \sqrt{3}}{4}$. So, the area formula for an equilateral triangle is $A = \frac{s^2 \sqrt{3}}{4}$.

47. The central angle of the pentagon is 72°. The base angles of the isosceles triangle formed by the radii of the pentagon is 54°. The apothem is $a = \frac{1}{2} \cdot s$ tan 54°.

\[ A = \frac{1}{2} \cdot a \cdot ns \]
\[ 72 = \frac{1}{2} \cdot a \cdot 5 \cdot s \]
\[ 144 = \left( \frac{1}{2} \cdot s \right) \text{tan} 54° \cdot s \]
\[ \frac{288}{5 \text{ tan} 54°} = s^2 \]
\[ s = \sqrt{\frac{288}{5 \text{ tan} 54°}} \approx 6.47 \]

The length of one side is about 6.47 centimeters.

48. The central angle of a dodecagon is 30° and the base angles of the isosceles triangle formed by the radii of the dodecagon is 15°. Find the apothem.

\[ \text{tan} 15° = \frac{\frac{1}{2}s}{a} \]
\[ a = \frac{\frac{1}{2}s \cdot \text{tan} 15°}{2 \cdot \text{tan} 15°} \]
\[ A = \frac{1}{2} \cdot a \cdot ns \]
\[ 140 = \frac{1}{2} \cdot a \cdot 12 \cdot s \]
\[ 280 = \left( \frac{s}{2 \text{ tan} 15°} \right) \cdot s \]
\[ \frac{280 \cdot 2 \cdot \text{tan} 15°}{12} = s^2 \]
\[ s = \sqrt{\frac{280 \cdot 2 \cdot \text{tan} 15°}{12}} \approx 3.54 \]

The length of one side is about 3.54 inches. So, the apothem is about $\frac{3.54}{2 \text{ tan} 15°} \approx 6.61$ inches.

49. Hexagon:

The apothem is

\[ \text{tan} 60° = \frac{a}{4} \]
\[ a = 6.9 \]

\[ A = \frac{1}{2} \cdot a \cdot \text{ns} \]
\[ \approx \frac{1}{2} \cdot 6.9 \cdot 6 \cdot 8 = 165.6 \]

Pentagon:

The central angle is $\frac{360°}{5} = 72°$. Since the triangle formed by the radii of the polygon is isosceles, each base angle is 54°. The base (side of the polygon) is divided into two equal parts, measuring 4. The apothem is

\[ \text{tan} 54° = \frac{a}{4} \]
\[ a = 4 \cdot \text{tan} 54° = 5.5 \]

\[ A = \frac{1}{2} \cdot a \cdot \text{ns} \]
\[ \approx \frac{1}{2} \cdot 5.5 \cdot 5 \cdot 8 = 110 \]

Square: $A = s^2 = 64$

Triangle:

The height is

\[ \text{tan} 60° = \frac{a}{4} \]
\[ a = 4 \cdot \text{tan} 60° = 6.9 \]

\[ A = \frac{1}{2} \cdot bh \]
\[ \approx \frac{1}{2} \cdot 8 \cdot 6.9 \]
\[ = 27.6 \]

Area of shaded region

\[ = \text{Area of hexagon} - \text{Area of pentagon} \]
\[ + \text{Area of square} - \text{Area of triangle} \]
\[ = 165.6 - 110 + 64 - 27.6 \]
\[ = 92 \]

The area of the shaded region is about 92 square units.

50. As $n$ approaches infinity, the $n$-gon approaches a circle.

As $n$ approaches infinity, $P$ approaches $\pi d$ or $2\pi r$, which is the circumference.

As $n$ approaches infinity, $a$ approaches $r$, the radius of the polygon and circle.

Substitute $r$ for $a$ and $C$ for $P$.

\[ A = \frac{1}{2} \cdot aP \]
\[ = \frac{1}{2} \cdot r \cdot C \]

So, the area of a circle is $\frac{1}{2} \cdot rC$. 

51. Pentagon:
The central angle is \( \frac{360^\circ}{5} = 72^\circ \). Since the triangle formed by the radii of the polygon is isosceles each base angle is 54°. The base (side of the polygon) is divided into two equal parts, measuring 25. The apothem is

\[
A = \frac{1}{2} a \cdot n s
\]

\[
= \frac{1}{2} \cdot 3.4 \cdot 5 \cdot 5
\]

\[= 42.5 \approx 43\]

Area of \( \triangle AEB \):
Because the triangle is isosceles and the vertex angle measures \((5 - 2) \cdot 180^\circ \div 5 = 3 \cdot 180^\circ \div 5 = 108^\circ \) and the base angles measure \((180^\circ - 108^\circ) \div 2 = 36^\circ \).

\[
sin 36^\circ = \frac{h}{5}
\]

\[h = 5 \sin 36^\circ \]

\[h \approx 2.94\]

\[
cos 36^\circ = \frac{x}{5}
\]

\[x = 5 \cos 36^\circ \]

\[x \approx 4.05\]

The base of the triangle is 2 \cdot 4.05 = 8.1.

\[
A = \frac{1}{2} bh
\]

\[
= \frac{1}{2} \cdot 8.1 \cdot 2.94
\]

\[= 11.9\]

Area of isosceles trapezoid \( EBCD \):
Find the height of the isosceles trapezoid.

\[
sin 72^\circ = \frac{h}{5}
\]

\[h = 5 \sin 72^\circ \]

\[h \approx 4.8\]

\[
A = \frac{1}{2} h (b_1 + b_2)
\]

\[
= \frac{1}{2} \cdot 4.8(8.1 + 5)
\]

\[= 31.44\]

Area of the pentagon
\[
= Area \ of \ \triangle AEB \ + \ Area \ of \ isosceles \ trapezoid \ EBCD
\]

\[= 11.9 + 31.44\]

\[= 43.34\]

Both methods yield approximately the same area, with a minor difference because of rounding decimal values.

*Sample answer:* The preferred method is using the formula

\[
A = \frac{1}{2} a \cdot ns
\]

because it involves fewer calculations.

52. Circumscribed polygon area

\[
= \frac{1}{2} a_{\text{out}} \cdot n_{\text{out}} s_{\text{out}}
\]

Inscribed polygon area

\[
= \frac{1}{2} a_{\text{in}} \cdot n_{\text{in}} s_{\text{in}}
\]

Large polygon:

\[
s_{\text{out}} = 2r \cdot \tan \left( \frac{180^\circ}{n} \right)
\]

\[
a_{\text{out}} = r
\]

\[
n_{\text{out}} = n
\]

\[
A = \frac{1}{2} r \cdot n \cdot 2r \cdot \tan \left( \frac{180^\circ}{n} \right)
\]

\[= r^2 \cdot n \cdot \tan \left( \frac{180^\circ}{n} \right)\]

Small polygon:

\[
s_{\text{in}} = 2r \cdot \sin \left( \frac{180^\circ}{n} \right)
\]

\[
a_{\text{in}} = r \cdot \cos \left( \frac{180^\circ}{n} \right)
\]

\[
n_{\text{in}} = n
\]

\[
A = \frac{1}{2} r \cdot \cos \left( \frac{180^\circ}{n} \right) \cdot n \cdot 2r \cdot \sin \left( \frac{180^\circ}{n} \right)
\]

\[= n \cdot r^2 \cdot \cos \left( \frac{180^\circ}{n} \right) \cdot \sin \left( \frac{180^\circ}{n} \right)\]

The difference in the areas of the two polygons is:

Circumscribed polygon area − Inscribed polygon area

\[
= r^2 \cdot n \cdot \tan \left( \frac{180^\circ}{n} \right) - n \cdot r^2 \cdot \cos \left( \frac{180^\circ}{n} \right) \cdot \sin \left( \frac{180^\circ}{n} \right)
\]

\[= r^2 \cdot n \left( \tan \left( \frac{180^\circ}{n} \right) - \cos \left( \frac{180^\circ}{n} \right) \cdot \sin \left( \frac{180^\circ}{n} \right) \right)\]

**Maintaining Mathematical Proficiency**

53. The figure has one vertical line of symmetry.

54. The figure has 4 lines of symmetry and rotational symmetry of 90°, 180°, and 270°.

55. The figure has rotational symmetry of 180°.

56. The figure has neither line nor rotational symmetry.

**11.4 Explorations (p. 617)**

<table>
<thead>
<tr>
<th>Solid</th>
<th>Vertices, ( V )</th>
<th>Edges, ( E )</th>
<th>Faces, ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>tetrahedron</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>cube</td>
<td>8</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>octahedron</td>
<td>6</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>dodecahedron</td>
<td>20</td>
<td>30</td>
<td>12</td>
</tr>
<tr>
<td>icosahedron</td>
<td>12</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

2. The number of vertices minus the number of edges plus the number of faces equal 2; \( V - E + F = 2 \).
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3. Sample answer:

- 5 vertices, 8 edges, 5 faces; \(5 - 8 + 5 = 2\)
- 10 vertices, 15 edges, 7 faces; \(10 - 15 + 7 = 2\)
- 6 vertices, 9 edges, 5 faces; \(6 - 9 + 5 = 2\)

11.4 Monitoring Progress (pp. 618–620)

1. Yes, the solid is a polyhedron. It is a square pyramid.

2. The solid is not a polyhedron. The face is an extended semicircle.

3. Yes, the solid is a polyhedron. It is a triangular prism.

4. The cross section is a pentagon.

5. The cross section is a hexagon.

6. The cross section is a circle.

7. The solid produced is a cone with height 4 and a base radius of 3.

8. The solid produced is a cylinder with height 8 and a base radius of 6.

9. The solid produced is a sphere with radius 7.

11.4 Exercises (pp. 621–622)

Vocabulary and Core Concept Check

1. A polyhedron is a solid that is bounded by polygons.

2. The cone does not belong because the others are pyramids.

Monitoring Progress and Modeling with Mathematics

3. B; The polyhedron is a rectangular pyramid.

4. D; The polyhedron is a pentagonal prism.

5. A; The polyhedron is a triangular prism.

6. C; The polyhedron is a hexagonal pyramid.

7. Yes, it is a polyhedron. It is a pentagonal pyramid.

8. Yes, it is a polyhedron. It is a hexagonal prism.

9. No, it is not a polyhedron.

10. Yes, it is a polyhedron. It is a trapezoidal prism.

11. The cross section is a circle.

12. The cross section is a square.

13. The cross section is a triangle.

14. The cross section is an octagon.
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15. The solid is a cylinder, with height 8 and a base radius 8.

16. The solid is a cone, with height 6 and base radius 6.

17. The solid is a sphere with radius 3.

18. The solid is a cylinder with height 5 and a base radius 2.

19. There are two parallel congruent bases, so it is a prism, not a pyramid. The base is a triangle, therefore it is a triangular prism.

20. Yes, the swimming pool is an octagonal prism.

21. Two parallel congruent bases, so it is a prism, not a pyramid. The base is a triangle, therefore it is a triangular prism.

22. Yes, the cross section could be a pentagon. The plane passes through five faces.

23. Yes, the cross section could be a rhombus. The plane passes through four faces and the side lengths are congruent.

24. Yes, the cross section can be an isosceles triangle. The plane passes through three edges from a common vertex and two points are the same distance from the vertex.

25. Your cousin is correct because the sides come together at a point.

26. The cross section is a rectangle.

27. The diagonal of the face of a cube is \( \sqrt{6^2 + 6^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2} \). The perimeter is \( 6 + 6\sqrt{2} + 6 + 6\sqrt{2} = 12 + 12\sqrt{2} \approx 28.97 \) inches.

28. a. The cross section is a rectangle.

29. No, a cross section cannot be a circle.

30. Yes, the cross section could be a pentagon. The plane passes through five faces.

31. Yes, the cross section could be a rhombus. The plane passes through four faces and the side lengths are congruent.

32. Yes, the cross section can be an isosceles triangle. The plane passes through three edges from a common vertex and two points are the same distance from the vertex.

33. Yes, the cross section can be a hexagon. The plane passes through six faces.
34. Yes, a cross section can be a scalene triangle. The plane passes through three edges from a common vertex and the three points are different distances from the vertex.

35. a. The composite solid is two cones with heights 3 and base radii 2.

b. The composite solid is a cylinder with height 8 and base radius 4. The top is a cone with height 3 and base radius 4.

36. Sample answer: At least three angles meet at a vertex, so the sum of the measures must be less than 360°. There can be three, four, or five triangles, three squares, or three pentagons at each vertex.

Maintaining Mathematical Proficiency
37. \( \triangle ABD \cong \triangle CDB \) by the SSS Triangle Congruence Theorem (Thm. 5.8).

38. \( \triangle JLK \cong \triangle JLM \) by the SAS Triangle Congruence Theorem (Thm. 5.5).

39. \( \triangle RQP \cong \triangle RTS \) by the ASA Triangle Congruence Theorem (Thm. 5.10).

11.1–11.4 What Did You Learn? (p. 623)
1. Sample answer: Each rotation of the wheel increases the total distance measured.

2. Sample answer: Is 12 the radius of the circle?

3. Sample answer: The original area is \( A = \frac{1}{2} d_1 d_2 \).
   Doubling the length of one diagonal produces \( \frac{1}{2} (2d_1)(d_2) = 2 \left( \frac{1}{2} d_1 d_2 \right) = 2A \), and doubling the lengths of both diagonals produces \( \frac{1}{2} (2d_1)(2d_2) = 4 \left( \frac{1}{2} d_1 d_2 \right) = 4A \).

11.1–11.4 Quiz (p. 624)
1. Arc length of \( \triangle H6114 EF \) is
   \[
   m \triangle H6114 EF = \frac{m \angle H6114 EF}{360°} \cdot 2\pi r
   \]
   \[
   \frac{13.7}{360°} \cdot 2\pi \approx 112.14°
   \]

2. Arc length of \( \triangle H6114 QS \) is
   \[
   m \triangle H6114 QS = \frac{m \angle H6114 QS}{360°} \cdot 2\pi r
   \]
   \[
   \frac{83°}{360°} \cdot 2\pi \approx 5.79
   \]

3. Arc length of \( \triangle H6114 LM \) is
   \[
   m \triangle H6114 LM = \frac{m \angle H6114 LM}{360°} \cdot C
   \]
   \[
   \frac{48°}{360°} \cdot C = \frac{8}{360°} \cdot 2\pi r
   \]
   \[
   C = 60
   \]

4. \( \angle RCY \) is a central angle; Therefore, \( m \angle RCY = \frac{360°}{8} = 45° \).
   Since \( \overline{CZ} \) is an apothem, \( \angle RCY \) is a central angle.

5. Area of sector \( \overarc{EF} \) is
   \[
   \text{Area} = \frac{m \angle H6114 EF}{360°} \cdot \pi \cdot r^2
   \]
   \[
   \approx 125.66
   \]

6. Area of sector \( \overarc{QS} \) is
   \[
   \text{Area} = \frac{m \angle H6114 QS}{360°} \cdot \pi \cdot r^2
   \]
   \[
   \approx 326.73
   \]

7. The center of \( RSTUVWXY \) is \( C \). \( CR \) and \( CY \) are radii of the octagon. \( CZ \) is an apothem. \( \angle RCY \) is a central angle.

8. \( \angle RCY \) is a central angle; Therefore, \( m \angle RCY = \frac{360°}{8} = 45° \).
   Since \( \overline{CZ} \) is an apothem, which makes it an altitude of isosceles \( \triangle RCY \), \( m \angle RCZ = 22.5° \) and \( m \angle ZRC = 67.5° \).
9. The central angle measures \( \frac{360°}{8} = 45° \). Since the triangle formed by the radii of the polygon is isosceles, each base angle measures 67.5°. Find the apothem.

\[
\sin 67.5° = \frac{a}{8} \\
a = 8 \sin 67.5° \approx 7.39
\]

Find the side length of the octagon.

\[
\cos 67.5° \approx \frac{x}{8} \\
x = 8 \cos 67.5° \approx 3.06
\]

So, the side length is about \( 2 \cdot 3.06 = 6.12 \).

\[
A = \frac{1}{2} a \cdot ns \\
\approx \frac{1}{2} \cdot 7.39 \cdot 8 \cdot 6.12 \approx 180.91
\]

The area of the regular octagon is above 180.91 square units.

11.5 Explorations (p. 625)

1. a. The volume of the prism is

\[
V = Bh = 2 \cdot 2 \cdot 8 = 32 \text{ cubic inches.}
\]

b. No, the volume has not changed because the amount of paper in the stack is the same.

c. Sample answer: If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

d. The height is still 8 inches and the dimensions of the paper are still 2 inches by 2 inches. Therefore, the volume is \( 2 \cdot 2 \cdot 8 = 32 \text{ cubic inches} \).

2. a. The base is a circle; Therefore the area of the circle is \( \pi r^2 = 4\pi \). The height is 3 inches.

\[
V = Bh \\
= 4\pi \cdot 3 \\
= 12\pi \approx 37.70 \text{ in}^2
\]

b. The area of the base is \( \pi r^2 = 25\pi \).

\[
V = Bh \\
= 25\pi \cdot 15 \\
= 375\pi \approx 1178.10 \text{ cm}^2
\]

3. The volume of a prism or cylinder is the product of the area of the base and the vertical height.

4. no; Sample answer: Each piece of paper would still have the same area.

11.5 Monitoring Progress (pp. 626–630)

1. The area of a base is

\[
B = \frac{1}{2} \cdot \frac{1}{2} \cdot 9 \cdot 5 = 22.5 \text{ m}^2
\]

and the height is \( h = 8 \text{ m} \).

\[
V = Bh \\
= 22.5 \cdot 8 \\
= 180
\]

The volume is 180 cubic meters.

2. \( V = \pi r^2 h \)

\[
= \pi (8)^2 (14) \\
= 896\pi \approx 2814.87
\]

The volume of the cylinder is about 2814.87 square feet.
Chapter 11

3. Change inches to centimeters.
   1 in. = 2.54 cm
   24 in. = 24 \cdot 2.54 = 60.96 cm
   32 in. = 32 \cdot 2.54 = 81.28 cm
   So, the radius is 60.96 centimeters and the height is 81.28 centimeters.

\[ V = \pi r^2 h \]
\[ = \pi (60.96)^2 (81.28) \approx 948,906.64 \]

**Density**

\[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} \]
\[ = \frac{2.3 \cdot x}{948,906.64} \approx 2,182,485 \]

The mass of the concrete cylinder is about 2,182,485 grams.

4. \[ V = Bh \]
   \[ 60 = 5 \cdot 3 \cdot h \]
   \[ 60 = 15 \cdot h \]
   \[ 4 = h \]
   The height should be 4 meters.

5. \[ V = Bh \]
   \[ 75 = B \cdot 5 \]
   \[ 15 = B \]
   The base is 15 square meters. Sample answer: A possible length is 5 meters and a possible width is 3 meters.

6. \[ \frac{\text{Volume}_C}{\text{Volume}_D} = \left( \frac{\text{Width}_C}{\text{Width}_D} \right)^3 \]
   \[ \frac{1536}{64} = \left( \frac{12}{3} \right)^3 \]
   \[ = 64 \]
   \[ \frac{1536}{64} = \text{Volume}_D \]
   \[ \text{Volume}_D = 24 \]
   The volume of prism D is 24 cubic meters.

7. **Volume of the larger triangular prism:**
   \[ A = \frac{1}{2}bh \]
   \[ = \frac{1}{2} \cdot 5\sqrt{3} \cdot 10 \]
   \[ = 25\sqrt{3} \]
   \[ V = bh \]
   \[ = 25\sqrt{3} \cdot 6 \]
   \[ = 150\sqrt{3} = 259.81 \]
   Volume of the rectangular prism:
   \[ V = Bh \]
   \[ 54 = 3 \cdot 3 \cdot 6 \]
   Volume of composite solid:
   \[ = \text{Volume of triangular prism} - \text{Volume of rectangular prism} \]
   \[ = 259.81 - 54 = 205.81 \]
   The volume of the composite solid is about 205.81 cubic feet.

11.5 Exercises (pp. 631–634)

**Vocabulary and Core Concept Check**

1. The volume of a solid is measured in cubic units.

2. Density is the amount of mass that an object has in a given unit of volume.

**Monitoring Progress and Modeling with Mathematics**

3. The area of the base is
   \[ A = \frac{1}{2} h(b_1 + b_2) \]
   \[ = \frac{1}{2} \cdot 1.8 \cdot (2.3 + 1.2) \]
   \[ = 3.15 \]
   \[ V = Bh \]
   \[ = 3.15 \cdot 2 \]
   \[ = 6.30 \]
   The volume of the trapezoidal prism is 6.3 cubic centimeters.

4. The area of the base is \[ A = hw = 4 \cdot 2 = 8. \]
   \[ V = Bh \]
   \[ = 8 \cdot 1.5 \]
   \[ = 12 \]
   The volume of the rectangular prism is 12 cubic meters.

5. The area of the base is \[ A = \frac{1}{2}bh = \frac{1}{2} \cdot 7 \cdot 10 = 35. \]
   \[ V = Bh \]
   \[ = 35 \cdot 5 \]
   \[ = 175 \]
   The volume of the triangular prism is 175 cubic inches.
Chapter 11

6. The area of the base is \( A = \ell w = 6 \cdot 11 = 66 \).

\[ V = Bh \]
\[ = 66 \cdot 14 \]
\[ = 924 \]

The volume of the rectangular prism is 924 cubic meters.

7. \( V = \pi r^2 h \)
\[ = \pi (3)^2 (10.2) \]
\[ \approx 288.40 \]

The volume of the cylinder is about 288.4 cubic feet.

8. The radius is \( 26.8 \div 2 = 13.4 \).

\[ V = \pi r^2 h \]
\[ = \pi (13.4)^2 (9.8) \]
\[ \approx 5528.22 \]

The volume of the cylinder is about 5528.22 cubic centimeters.

9. \( V = \pi r^2 h \)
\[ = \pi (5)^2 (8) \]
\[ \approx 628.32 \]

The volume of the cylinder is about 628.32 cubic feet.

10. The radius is \( 12 \div 2 = 6 \). So, the area of the base is \( A = \pi r^2 = \pi (6)^2 = 36\pi \). The height is the longer leg of a 30°-60°-90° triangle, with a hypotenuse of 18.

hypotenuse = shorter leg \( \cdot 2 \)
\[ 18 = \text{shorter leg} \cdot 2 \]
\[ 9 = \text{shorter leg} \]

longer leg = shorter leg \( \cdot \sqrt{3} = 9\sqrt{3} \)

So, the height is \( 9\sqrt{3} \).

\[ V = Bh \]
\[ = 36\pi \cdot 9\sqrt{3} \]
\[ \approx 1763.01 \]

The volume of the cylinder is about 1763.01 cubic meters.

11. The base is an equilateral triangle. The height of the equilateral triangle with a side of 8 is
\[ \sqrt{8^2 - 4^2} = \sqrt{48} = 4\sqrt{3} \]. So, the area of the base is
\[ A = \frac{1}{2}bh = \frac{1}{2} \cdot 4\sqrt{3} \cdot 8 = 16\sqrt{3} \].

\[ V = Bh \]
\[ = 16\sqrt{3} \cdot 4\sqrt{3} \]
\[ \approx 310.38 \]

The volume of the triangular prism is about 310.38 cubic centimeters.

12. Area of the pentagonal base:
The central angle is \( \frac{360^\circ}{5} = 72^\circ \). Since the triangle formed by the radii of the polygon is isosceles, each base angle is 54°. The base (side of the polygon) is divided into two equal parts, 1.5. The apothem is
\[ \tan 54^\circ = \frac{a}{1.5} \]
\[ a = 1.5 \cdot \tan 54^\circ \approx 2.06 \].

\[ A = \frac{1}{2} a \cdot ns \]
\[ \approx \frac{1}{2} \cdot 2.06 \cdot 5 \cdot 3 = 15.45 \]

\[ V = Bh \]
\[ \approx 15.45 \cdot 9 \]
\[ = 139.05 \]

The volume of the pentagonal prism is about 139.05 cubic feet.
Chapter 11

13. Density = \( \frac{\text{Mass}}{\text{Volume}} \)
   Copper: Density = \( \frac{73.92}{8.25} = 8.96 \text{ g/cm}^3 \)
   Iron: Density = \( \frac{39.35}{5} = 7.87 \text{ g/cm}^3 \)
   Copper has the greater density.

14. The radius of the coin is \( 40.6 \div 2 = 20.3 \).
   \[ V = \pi r^2 h \]
   \[ = \pi (20.3)^2 (2.98) \]
   \[ \approx 3857.96 \]
   The volume of the coin is about 3857.96 cubic millimeters or 3.86 cubic centimeters.

15. The base circumference was used instead of the base area.
   \[ V = \pi r^2 h \]
   \[ = \pi \cdot 4^2 \cdot 3 \]
   \[ = 48\pi \]
   The volume of the cylinder is 48\pi cubic feet.

16. The density formula was set up incorrectly.
   \[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} \]
   \[ 10.5 = \frac{x}{3.86} \]
   \[ 10.5 \cdot 3.86 = x \]
   \[ x = 40.53 \]
   The mass of the coin is about 41 grams.

17. \( V = Bh \)
   560 = 7 \cdot 8 \cdot u
   560 = 56u
   u = 10
   The height is 10 feet.

18. \( V = Bh \)
   2700 = 12 \cdot 15 \cdot v
   2700 = 180v
   v = 15
   The height is 15 yards.

19. The area of the base is \( A = \frac{1}{2} bh = \frac{1}{2} \cdot 5 \cdot 8 = 20 \).
   Volume of a triangular prism = Area of the base \cdot \text{height}
   \[ V = Bh \]
   \[ 80 = 20 \cdot w \]
   \[ w = 4 \]
   The height of the prism is 4 centimeters.

20. Area of the hexagonal base:
   The central angle is \( \frac{360°}{6} = 60° \). Since the triangle formed by the radii of the polygon is isosceles, each base angle is 60°. The base (side of the polygon) is divided into two equal parts, each equal to 1. The apothem is the shorter leg \( \cdot \sqrt{3} \). Therefore, \( a = \sqrt{3} \).
   \[ A = \frac{1}{2} a \cdot ns \]
   \[ = \frac{1}{2} \cdot \sqrt{3} \cdot 6 \cdot 2 \]
   \[ = 6\sqrt{3} \]
   \[ V = Bh \]
   \[ 72.66 = 6\sqrt{3} \cdot x \]
   \[ 6.99 = x \]
   The height of the prism is about 6.99 inches.

21. \( V = \pi r^2 h \)
   3000 = \pi \cdot 9.3^2 \cdot y
   11.04 = y
   The height of the cylinder is about 11.04 feet.

22. \( V = \pi r^2 h \)
   1696.5 = \pi \cdot z^2 \cdot 15
   \[ z^2 = 36 \]
   \[ y = 6.00 \]
   The radius of the cylinder is about 6 meters.

23. \( V = Bh \)
   154 = B \cdot 11
   14 = B
   The area of the base is 14 square inches. Sample answer: Possible lengths and widths are 2 inches by 7 inches or 3.5 inches by 4 inches.

24. \( V = Bh \)
   27 = B \cdot 3
   9 = B
   The area of the base is 9 square meters. Sample answer: A possible length and width of the base are 3 meters by 3 meters.
Chapter 11

25. \[ \text{Volume}_A = \left( \frac{\text{Height}_A}{\text{Radius}_A} \right)^3 \]
\[
\frac{2673}{27} = \left( \frac{9}{3} \right)^3
\]

The volume of solid B is 99 cubic centimeters.

26. \[ \text{Volume}_A = \left( \frac{\text{Side}_A}{\text{Radius}_A} \right)^3 \]
\[
\frac{4608\pi}{64} = \left( \frac{12}{5} \right)^3
\]

The volume of solid B is about 28,274 cubic centimeters.

27. \[ \text{Volume}_A = \left( \frac{\text{Height}_A}{\text{Height}_B} \right)^3 \]
\[
\frac{12}{40.5} = \left( \frac{x}{3} \right)^3
\]

The side length of prism A is 2 centimeters.

28. \[ \text{Volume}_A = \left( \frac{\text{Height}_A}{\text{Height}_B} \right)^3 \]
\[
\frac{7\pi}{56\pi} = \left( \frac{5}{h} \right)^3
\]

The side height of cylinder B is 10 feet.

29. The volume of the top prism is \(5 \cdot 3 \cdot 2 = 30\) cubic feet.

The volume of the bottom prism is \(10 \cdot 6 \cdot 2 = 120\) cubic feet.

The volume of the composite solid is 150 cubic feet.

30. The volume of the half cylinder is \(\frac{1}{2} \cdot \pi \cdot 2^2 \cdot 4 = 8\pi\).

The volume of the cube is \(4 \cdot 4 \cdot 4 = 64\). Volume of the half cylinder + Volume of the cube = \(8\pi + 64 \approx 89.13\)

The volume of the composite solid is about 89.13 cubic inches.

31. The volume of the larger cylinder is \(\pi \cdot 8^2 \cdot 11 = 704\pi\).

The volume of the smaller cylinder is \(\pi \cdot 3^2 \cdot 11 = 99\pi\).

Volume of the larger cylinder – Volume of the smaller cylinder = \(704\pi - 99\pi = 605\pi = 1900.66\)

The volume of the composite solid is about 1900.66 cubic inches.

32. The volume of the larger prism is \(4 \cdot 2 \cdot 5 = 40\).

The volume of the inner prism is \(1 \cdot 1 \cdot 5 = 5\).

Volume of the larger prism – Volume of the inner prism = \(40 - 5 = 35\)

The volume of the composite solid is 35 cubic feet.

33. \(V = \pi r^2 h\)

\(= \pi \cdot (500)^2 \cdot 400 \approx 100,000,000\pi\)

Amount of water:

1 cubic foot = 7.48 gal

100,000,000\pi cubic feet = \(2.35 \times 10^9\) gallons

The Great Blue Hole contains about \(2.35 \times 10^9\), or \(2,350,000,000\) gallons of water.

34. The volume of the block is \(1.31 \cdot 0.66 \cdot 0.66 = 0.57\).

The volume of the smaller holes is \(2 \cdot (0.33 \cdot 0.66 \cdot 0.39) = 0.17\).

Volume of the larger block

– Volume of the two smaller blocks

= \(0.57 - 0.17 = 0.40\)

The volume of the solid is about 0.40 cubic feet.

Sample answer: The formula for the volume of a prism is slightly less complicated.

35. The volume of the block of wax is \(10 \cdot 9 \cdot 20 = 1800\) cubic centimeters.

The cylinder candle has a volume of \(\pi (4.5)^2 \cdot 12 = 763.4\) cubic centimeters.

\(1800 = 2.36 \approx 2\)

763.4

So, 2 cylindrical candles with a diameter of 9 centimeters and height of 12 centimeters can be made.
Chapter 11

36. The volume of the triangular prism is 
\[ \frac{1}{2} \cdot 6 \cdot 8 \cdot 10 = 240 \] cubic centimeters.

\[ 1800 = 7.5 \approx 7 \]

So, 7 candles with the shape of a triangular prism with dimensions 8 by 6 centimeters and height of 10 centimeters can be made.

37. The volume of the aquarium will contain
\[ \frac{3}{4} (30 \cdot 10 \cdot 15) = 4500 \] cubic inches of water.

a. The volume changes when the rock is submerged to
\[ \frac{3}{4} (30 \cdot 10 \cdot 15.25) = 4575 \] cubic inches.

\[ 4575 - 4500 = 75 \]

So, the volume of the rock is 75 cubic inches.

b. The volume of the aquarium completely filled is
\[ 2 \cdot 10 \cdot 1 + 2 \cdot 10 \cdot 1 + 2 \cdot 10 \cdot 10 = 240 \] square inches.

The dimensions of a second rectangular prism are
\[ 25 \times 2 \times 2 \]. The volume is \( 25 \cdot 2 \cdot 2 = 100 \) cubic inches.

The surface area is
\[ 2 \cdot 25 \cdot 2 + 2 \cdot 2 + 2 + 2 \cdot 25 \cdot 2 = 208 \] square inches.

43. Sample answer: The dimensions of one rectangular prism are
\[ 10 \times 1 \times 10 \]. The volume is \( 10 \cdot 1 \cdot 10 = 100 \) cubic inches.

The surface area is
\[ 2 \cdot 10 \cdot 1 + 2 \cdot 10 \cdot 1 + 2 \cdot 10 \cdot 10 = 240 \] square inches.

The dimensions of a second rectangular prism are
\[ 25 \times 2 \times 2 \]. The volume is \( 25 \cdot 2 \cdot 2 = 100 \) cubic inches.

The surface area is
\[ 2 \cdot 25 \cdot 2 + 2 \cdot 2 + 2 + 2 \cdot 25 \cdot 2 = 208 \] square inches.

The solid produced by rotating around the vertical line has a greater volume.

44. The volume of the $6 box of cereal is
\[ 16 \cdot 4 \cdot 10 = 640 \] cubic inches.

The cost per cubic inch for this box is \[ \frac{600}{640} = 0.94 \text{ cents per in.}^3 \].

The volume of the $2 box of cereal is
\[ 2 \cdot 8 \cdot 10 = 160 \] cubic inches.

The cost per cubic inch for this box is \[ \frac{200}{160} = 1.25 \text{ cents per in.}^3 \].

The $6 box of cereal will give you more for your money. The cost per cubic inch is less than the cost of the $2 box.

45. Solid produced by rotating around horizontal line:
\[ V = \pi r^2 h \]
\[ = \pi \cdot 3^2 \cdot 5 \]
\[ = 45\pi \text{ in.}^3 \]

Solid produced by rotating around vertical line:
\[ V = \pi r^2 h \]
\[ = \pi \cdot 5^2 \cdot 3 \]
\[ = 75\pi \text{ in.}^3 \]

The solid produced by rotating around the vertical line has a greater volume.
Chapter 11

46. \( h_Y = 2h_V \)
   \( r_Y = \frac{1}{2}r_X \)
   \( V_Y = \pi r_Y^2 h_Y \)
   \( V_X = \pi r_Y^2 h_Y \)
   \( V_X = \pi \left( \frac{1}{2}r_Y \right)^2 \cdot (2h_Y) \)
   \( V_X = \frac{1}{4} \cdot 2 \cdot \pi \cdot r_Y^2 \cdot h_Y \)
   \( V_X = \frac{1}{2} \cdot \pi \cdot r_Y^2 \cdot h_Y \)
   \( V_X = \frac{1}{2} \cdot V_Y \)

The volume of cylinder \( X \) is half the volume of cylinder \( Y \).

Doubling the height doubles the volume and halving the radius multiplies the volume by \( \frac{1}{4} \).

47. Length of sector = \( \frac{60^\circ}{360^\circ} \cdot 2 \cdot \pi \cdot r \)

   \[
   \frac{2}{3} \pi = \frac{1}{6} \cdot 2 \cdot \pi \cdot r \\
   2 = r
   \]

Area of sector = \( \frac{60^\circ}{360^\circ} \cdot \pi \cdot 2^2 \)

\[
= \frac{2}{3} \pi
\]

\( V = Bh = \frac{2}{3} \cdot \pi \cdot 3.5 \approx 7.33 \)

The volume of the solid is about 7.33 cubic inches.

48. The volume of the larger cylinder is \( V = \pi R^2 h \).

The volume of the smaller cylinder is \( V = \pi r^2 h \).

\[
\pi r^2 h = \frac{1}{2} \pi R^2 h
\]

\[
2r^2 = R^2
\]

\[
R = r\sqrt{2}
\]

\[
\frac{R}{\sqrt{2}} = r
\]

\[
\frac{\sqrt{2}}{2} \cdot R = r
\]

\[
\frac{R\sqrt{2}}{2} = r
\]

49. Increase the height by 25%.

50. \( V = s^3 \)

   \[
   0.6 \cdot V = x \cdot s^3
   \]

   \[
   0.6s^3 = (xs)^3
   \]

   \[
   x^3s^3 = 0.6s^3
   \]

   \[
   x = \sqrt[3]{0.6}
   \]

   \[
   1 - \frac{3}{0.6} = 0.1566
   \]

So, reduce the edge length by about 15.66%.

51. yes; Sample answer: Density is proportional to mass when the volume is constant.

52. no; Sample answer: The bases are the same but the sides of the other faces change.

53. The area of the pentagon is the area of the rectangular base,

   \[
   18 \cdot 8 = 144 \text{ square feet, and the area of the triangular top,}
   \]

   \[
   \frac{1}{2} \cdot 18 \cdot (\sqrt{3^2 - 9^2}) \text{. So, the total area of the pentagonal base}
   \]

   \[
   144 + 9 \cdot \sqrt{3^2 - 9^2} \text{ square feet.}
   \]

   \[
   V = Bh
   \]

   \[
   = (144 + 9 \cdot \sqrt{3^2 - 81}) \cdot 36
   \]

9072 = \( (144 + 9 \cdot \sqrt{3^2 - 81}) \cdot 36 \)

252 = 144 + 9 \cdot \sqrt{3^2 - 81}

12 = \sqrt{3^2 - 81}

144 = \( x^2 - 81 \)

225 = \( x^2 \)

\[
x = 15
\]

The slanted edges of the roof have a length of 15 feet. So, the dimensions of each half of the roof are 15 feet by 36 feet.

54. To find the side of the outside top use \( x = \frac{4}{3 \tan 54^\circ} \).

So, half of the side is about 2.91 centimeters and the side is about 5.82 centimeters.

The volume of the outside of the box is

\[
\frac{1}{2} \cdot 4 \cdot 5 \cdot 5.82 \cdot 6 = 349.2 \text{ cubic centimeters.}
\]

Since the apothem of the outside box is 4, the apothem of the inside box is 3 centimeters.

To find the side of the inside top use \( x = \frac{3}{3 \tan 54^\circ} \).

So, half of the side is about 2.18 centimeters and the side is about 4.36 centimeters. The height of the inside of the box is \( 6 - 2 = 4 \) centimeters.

The volume of the inside of the box is

\[
\frac{1}{2} \cdot 3 \cdot 5 \cdot 4.36 \cdot 4 = 130.8 \text{ cubic centimeters.}
\]

Volume of the outside + Volume of the inside + Volume of the wood

\[
= 349.2 - 130.8 = 218.40
\]

The approximate volume of wood needed to construct the box is 218.4 cubic centimeters.

Maintaining Mathematical Proficiency

55. The surface area of the pyramid is the area of each face plus the area of the base.

Face area = \( \frac{1}{2} \cdot 3 \cdot 2 = 3 \)

Area of the base = 4

Total surface area = 4 \cdot Face area + Base area

\[
= 4 \cdot 3 + 4 = 12 + 4 = 16
\]

The surface area of the regular pyramid is 16 square meters.
Chapter 11

56. The surface area of the pyramid is the area of each face plus the area of the base.

Face area = \( \frac{1}{2} \cdot 10 \cdot 8 = 40 \)

Area of the base = 166.3

Total surface area = 6 \cdot \text{Face area} + \text{Base area}
\[= 6 \cdot 40 + 166.3 = 240 + 166.3 = 406.3\]

The surface area of the regular pyramid is 406.3 square centimeters.

57. The surface area of the pyramid is the area of each face plus the area of the base.

Face area = \( \frac{1}{2} \cdot 20 \cdot 18 = 180 \)

Area of the base = \( \frac{1}{2} \cdot 15.6 \cdot 18 = 140.4 \)

Total surface area = 3 \cdot \text{Face area} + \text{Base area}
\[= 3 \cdot 180 + 140.4 = 540 + 140.4 = 680.4\]

The surface area of the regular pyramid is 680.4 square inches.

11.6 Explorations (p. 635)

1. The volume of a pyramid is
\[
\frac{1}{3} \cdot \text{area of the base} \cdot \text{height},
\]

\[
V = \frac{1}{3}bh.
\]

2. Find the apothem by using a 30°-60°-90° triangle. The apothem is the longer leg, opposite the 60° angle. It equals the shorter side times \( \sqrt{3} \); therefore it has length \( \sqrt{3} \). The area of the hexagonal base is \( \frac{1}{2} \cdot \sqrt{3} \cdot 2 \cdot 6 = 6 \cdot \sqrt{3} \). So, the volume is about \( \frac{1}{3} \cdot 6 \cdot \sqrt{3} \cdot 3 = 6\sqrt{3} = 10.39 \) cubic inches.

3. Use the formula \( V = \frac{1}{3}Bh \).

4. Sample answer: Use a cone and cylinder with the same height and circular base and determine how many cones of sand are needed to fill cylinder.

11.6 Monitoring Progress (pp. 637–638)

1. \( V = \frac{1}{3}Bh \)
\[= \frac{1}{3}(10^2)(12) = 400 \]

The volume of the pyramid is 400 cubic centimeters.

2. The apothem equals the shorter leg \( \cdot \sqrt{3} \). Therefore,
\[a = 6\sqrt{3}.
\]

\[
A = \frac{1}{2}a \cdot ns = \frac{1}{2} \cdot 6\sqrt{3} \cdot 6 \cdot 12 = 216\sqrt{3}
\]

\[
V = \frac{1}{3}Bh = \frac{1}{3} \cdot 216\sqrt{3} \cdot 20 = 1440\sqrt{3} = 2494.15
\]

The volume of the pyramid is about 2494.15 cubic centimeters.

3. \( V = \frac{1}{3}Bh \)
\[75 = \frac{1}{3} \cdot B \cdot 9
\]
\[75 = 3B
\]
\[25 = B
\]

The area of the base is 25. Since the base is a square, the side length is \( \sqrt{25} = 5 \) meters.

4. \( V = \frac{1}{3}Bh \)
\[24 = \frac{1}{3}(\frac{1}{2} \cdot 6 \cdot 3)h
\]
\[24 = 3h
\]
\[8 = h
\]

The height of the pyramid is 8 meters.

5. The scale factor is \( K = \frac{\text{Side length of pyramid D}}{\text{Side length of pyramid C}} = \frac{3}{9} = \frac{1}{3} \)

Volume of pyramid D
\[
\text{Volume of pyramid C} = K^3
\]

\[
\text{Volume of pyramid D} = \frac{324}{1^3} = 324
\]

Volume of pyramid D = 12

The volume of pyramid D is 12 cubic meters.

6. The volume of the bottom of the solid is \( \frac{1}{2} \cdot 4 \cdot 8 \cdot 5 = 80 \).

The volume of the top of the solid is \( \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 4 \cdot 8 \cdot 3 \right) = 16 \).

Volume of composite solid = Top + Bottom
\[= 80 + 16 = 96
\]

The volume of the composite solid is 96 cubic feet.

11.6 Exercises (pp. 639–640)

Vocabulary and Core Concept Check

1. Sample answer: A triangular prism has two parallel bases that are triangles. A triangular pyramid has one base that is a triangle, and the other faces all intersect at a single point.
2. The volume of the square pyramid is \( \frac{1}{3} \) of the volume of the cube.

**Monitoring Progress and Modeling with Mathematics**

3. \( V = \frac{1}{3}Bh \)
   \[ = \frac{1}{3} \cdot (12 \cdot 16) \cdot 7 \]
   \[ = \frac{1}{3} \cdot (192) \cdot 7 \]
   \[ = 448 \]
   The volume of the pyramid is 448 cubic meters.

4. \( V = \frac{1}{3}Bh \)
   \[ = \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 3 \cdot 4 \right) \cdot 3 \]
   \[ = \frac{1}{3} \cdot (3 \cdot 2) \cdot 3 \]
   \[ = 6 \]
   The volume of the pyramid is 6 cubic inches.

5. \( V = \frac{1}{3}Bh \)
   \[ 120 = \frac{1}{3} \cdot B \cdot 10 \]
   \[ 360 = 10B \]
   \[ 36 = B \]
   The area of the square base is 36 square meters. So, the side length is \( \sqrt{36} = 6 \) meters.

6. \( V = \frac{1}{3}Bh \)
   \[ 912 = \frac{1}{3} \cdot B \cdot 19 \]
   \[ 2736 = 19B \]
   \[ 144 = B \]
   The area of the square base is 144 square feet. So, the side length is \( \sqrt{144} = 12 \) feet.

7. \( V = \frac{1}{3}Bh \)
   \[ V = \frac{1}{3} \cdot (\ell \cdot w) \cdot h \]
   \[ 480 = \frac{1}{3} \cdot (\ell \cdot 9) \cdot 10 \]
   \[ 480 = 30\ell \]
   \[ 16 = \ell \]
   The side length of the rectangular base is 16 inches.

8. \( V = \frac{1}{3}Bh \)
   \[ V = \frac{1}{3} \cdot (\ell \cdot w) \cdot h \]
   \[ 105 = \frac{1}{3} \cdot (7 \cdot w) \cdot 15 \]
   \[ 105 = 35w \]
   \[ 3 = w \]
   The width of the rectangular base is 3 centimeters.

9. One side length was used in the formula as the base area.
   \[ V = \frac{1}{3}(6)(5) \]
   \[ = \frac{1}{3}(36)(5) \]
   \[ = 60 \text{ ft}^3 \]

10. Sample answer: A rectangular pyramid with a base area of 5 square meters and a height of 6 meters, and a rectangular prism with a base area of 5 square meters and a height of 2 meters; Both volumes are 10 cubic meters.

11. \( V = \frac{1}{3}Bh \)
   \[ 15 = \frac{1}{3} \cdot (3 \cdot 3) \cdot h \]
   \[ 15 = 3h \]
   \[ 5 = h \]
   The height of the pyramid is 5 feet.

12. \( V = \frac{1}{3}Bh \)
   \[ 224 = \frac{1}{3} \cdot (8 \cdot 12) \cdot h \]
   \[ 224 = 32 \cdot h \]
   \[ 7 = h \]
   The height of the pyramid is 7 inches.

13. \( V = \frac{1}{3}Bh \)
   \[ 198 = \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 11 \cdot 9 \right) \cdot h \]
   \[ 198 = 16.5 \cdot h \]
   \[ 12 = h \]
   The height of the pyramid is 12 yards.

14. \( V = \frac{1}{3}Bh \)
   \[ V = \frac{1}{3} \cdot \left( \frac{1}{2} \cdot w \right) \cdot h \]
   \[ 392 = \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 7 \cdot 14 \right) \cdot h \]
   \[ 1176 = 49 \cdot h \]
   \[ 24 = h \]
   The height of the pyramid is 24 centimeters.
Chapter 11

15. The scale factor is \( K = \frac{\text{Height of pyramid B}}{\text{Height of pyramid A}} = \frac{3}{12} = \frac{1}{4} \).

Volume of pyramid B = \( K^3 \)

Volume of pyramid A = \( \frac{1^3}{256} \)

Volume of pyramid B = 4

The volume of pyramid B is 4 cubic feet.

16. The scale factor is \( K = \frac{\text{Side length of pyramid B}}{\text{Side length of pyramid A}} = \frac{6}{3} = 2 \).

Volume of pyramid B = \( K^3 \)

Volume of pyramid A = \( \frac{(1)^3}{10} \)

Volume of pyramid B = 80

The volume of pyramid B is 80 cubic inches.

17. The volume of the bottom of the solid is \( 2 \cdot 4 \cdot 6 = 48 \).

The volume of the top of the solid is \( \frac{1}{3} \cdot (4 \cdot 6) \cdot 3 = 24 \).

Volume of composite solid = Top + Bottom = \( 48 + 24 = 72 \)

The volume of the composite solid is 72 cubic inches.

18. The volume of the bottom of the solid is \( \frac{1}{2} \cdot (12 \cdot 9) \cdot 10 = 540 \).

The volume of the top of the solid is \( \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 12 \cdot 9 \right) \cdot 7 = 126 \).

Volume of composite solid = Top + Bottom = \( 540 + 126 = 666 \)

The volume of the composite solid is 666 cubic centimeters.

19. The volume of the bottom of the solid is \( \frac{1}{3} \cdot (8 \cdot 8) \cdot 5 \approx 106.67 \).

The volume of the top of the solid is \( \frac{1}{3} \cdot (8 \cdot 8) \cdot 5 = 106.67 \).

Volume of composite solid = Top + Bottom

\( \approx 106.67 + 106.67 \)

\( = 213.33 \)

The volume of the composite solid is about 213.33 cubic centimeters.

20. The volume of the outside solid is \( 12 \cdot 12 \cdot 12 = 1728 \).

The volume of the inside solid is \( \frac{1}{3} \cdot (6\sqrt{2} \cdot 6\sqrt{2}) \cdot 12 = 288 \).

Volume of composite solid = Outside + Inside

\( = 1728 + 288 \)

\( = 1440 \)

The volume of the composite solid is 1440 cubic inches.

21. \( V = \frac{1}{3} Bh = \frac{1}{3} \cdot (6 \cdot 6) \cdot 8 = 96 \text{ ft}^3 \)

\( a. \) Double the height, so \( h = 2 \cdot 8 = 16 \).

\( V = \frac{1}{3} \cdot (36) \cdot 16 = 192 = 2 \cdot 96 \text{ ft}^3 \)

The volume doubles.

\( b. \) Double the side length of the base, so \( B = (2 \cdot 6)^2 = 144 \).

\( V = \frac{1}{3} \cdot (144) \cdot 8 = 384 = 4 \cdot 96 \text{ ft}^3 \)

The volume is 4 times greater.

\( c. \) \( V = \frac{1}{3} \cdot s^2 \cdot h \)

Replace \( h \) with \( 2h \).

\( V = \frac{1}{3} \cdot s^2 \cdot 2h \)

\( = 2 \cdot \frac{1}{3} \cdot s^2 \cdot h \)

\( = 2 \cdot V \)

\( V = \frac{1}{3} \cdot s^2 \cdot h \)

Replace \( s \) with \( 2s \).

\( V = \frac{1}{3} \cdot (2s)^2 \cdot h \)

\( = \frac{1}{3} \cdot 4s^2 \cdot h \)

\( = 4 \cdot \frac{1}{3} \cdot s^2 \cdot h \)

\( = 4 \cdot V \)

The answers in parts (a) and (b) are true for any square pyramid. Doubling the height doubles the volume. Doubling the side length of the base quadruples the volume.

22. Sample answer: The three pyramids have the same base and height as the prism, and the same volumes as each other, so each is \( \frac{1}{3} \) the volume of the prism.
Chapter 11

23. Area of the pentagon:

The central angle is \(\frac{360^\circ}{5} = 72^\circ\). Since the triangle formed by the radii of the polygon is isosceles, each base angle is \(54^\circ\). The base (side of the polygon) is divided into two equal parts, 1.5 each. Find the apothem.

\[
tan \, 54^\circ = \frac{a}{1.5}
\]

\[
a = 1.5 \cdot \tan \, 54^\circ \approx 2.065
\]

Find the radius of the pentagon.

\[
sin \, 54^\circ = \frac{a}{r}
\]

\[
sin \, 54^\circ = \frac{2.065}{r}
\]

\[
r = \frac{2.065}{\sin \, 54^\circ} \approx 2.55
\]

\[
A = \frac{1}{2}a \cdot ns
\]

\[
A \approx \frac{1}{2} \cdot 2.065 \cdot 5 \cdot 3 = 15.49
\]

Height of the pyramid:

\[
tan \, 35^\circ = \frac{h}{2.5}
\]

\[
h = 2.5 \cdot \tan \, 35^\circ \approx 1.79
\]

Volume of the pyramid:

\[
V = \frac{1}{3} \cdot (15.49) \cdot 1.79 \approx 9.24
\]

The volume of the regular pentagonal pyramid is about 9.24 cubic feet.

24. Let the missing part of the pyramid have a height of \(h_1\).

Volume of the top of the pyramid: \(V = \frac{1}{3}bh_1\)

Volume of the whole pyramid: \(V = \frac{1}{3}a^2(h_1 + h)\)

Volume of the frustum = Volume of whole pyramid − Volume of the top of the pyramid

\[
V_f = \frac{1}{3}a^2(h_1 + h) - \frac{1}{3}bh_1
\]

\[
V_f = \frac{1}{3}a^2h_1 + \frac{1}{3}a^2h - \frac{1}{3}bh_1
\]

\[
V_f = \frac{1}{3}a^2h_1 - \frac{1}{3}b^2h_1 + \frac{1}{3}a^2h
\]

\[
V_f = \frac{1}{3}(a^2h_1 - b^2h_1 + a^2h)
\]

\[
V_f = \frac{1}{3}[h_1(a^2 - b^2) + a^2h]
\]

\[
V_f = \frac{1}{3}[a^2h + h_1(a^2 - b^2)]
\]

Using similar triangles:

\[
h_1 = \frac{b}{h + h_1}
\]

Solve for \(h_1\).

\[
ah_1 = bh + bh_1
\]

\[
h_1(a - b) = bh
\]

\[
h_1 = \frac{bh}{a - b}
\]

Substitute for \(h_1\) in \(V_f\)

\[
V_f = \frac{1}{3}[a^2h + \left(\frac{bh}{a - b}\right)(a^2 - b^2)]
\]

\[
V_f = \frac{1}{3}[a^2h + \left(\frac{bh}{a - b}\right)(a - b)(a + b)]
\]

\[
V_f = \frac{1}{3}(a^2h + bh(a + b))
\]

\[
V_f = \frac{1}{3}a^2h + \frac{1}{3}bh^2 + \frac{1}{3}abh + \frac{1}{3}b^2h]
\]

25. Find the volume of the hexagonal prism with side length 3.5 inches and height 1.5 inches.

Apothem: \(\tan \, 60^\circ = \frac{a}{1.75}\)

\[
a \approx 3.03
\]

\[
A = \frac{1}{2}a \cdot ns
\]

\[
A \approx \frac{1}{2} \cdot 3.03 \cdot 6 \cdot 3.5 \approx 31.82
\]

The volume of the first hexagonal prism is about 31.82 \cdot 1.5 = 47.72 cubic inches.

Find the volume of the second hexagonal prism with side length 3.25 inches and height 0.25 inches.

Apothem: \(\tan \, 60^\circ = \frac{a}{1.63}\)

\[
a \approx 2.81
\]

\[
A = \frac{1}{2}a \cdot ns
\]

\[
A \approx \frac{1}{2} \cdot 2.81 \cdot 6 \cdot 3.25 \approx 27.40
\]

The volume of the second hexagonal prism is about 27.40 \cdot 0.25 = 6.85 cubic inches.

Find the volume of the hexagonal pyramid with side length 3 inches and height 3 inches.

Apothem: \(\tan \, 60^\circ = \frac{a}{1.5}\)

\[
a \approx 2.60
\]

\[
A = \frac{1}{2}a \cdot ns
\]

\[
A \approx \frac{1}{2} \cdot 2.60 \cdot 6 \cdot 3 = 23.4
\]

The volume of the hexagonal pyramid is about \(\frac{1}{3} (23.4)(3) = 23.4\) cubic inches.

The volume of the Nautical deck prism is about 47.72 + 6.85 + 23.4 = 77.97 cubic inches.

**Maintaining Mathematical Proficiency**

26. \(\tan \, 35^\circ = \frac{x}{9}\)

\[
x = \frac{9}{\tan \, 35^\circ}
\]

\[
x \approx 12.9
\]
Chapter 11

27. \( \sin 57^\circ = \frac{x}{15} \)
   \( 15 \cdot \sin 57^\circ = x \)
   \( 12.6 \approx x \)

28. longer leg = shorter leg \( \cdot \sqrt{3} \)
   \( 10 = x\sqrt{3} \)
   \( \frac{10}{\sqrt{3}} = x \)
   \( x = \frac{10\sqrt{3}}{3} \approx 5.8 \)

29. \( \cos 64^\circ = \frac{7}{x} \)
   \( x = \frac{7}{\cos 64^\circ} \)
   \( x \approx 16.0 \)

11.7 Explorations (p. 641)

1. a. Sample answer: The points on the base are all the same distance from the point on the same plane directly below the vertex of the cone.
   The circumference is \( 5\pi \) inches. The radius is \( \frac{5}{2} = 2.5 \) inches.

   b. The area of the original circle is \( \pi r^2 = \pi (3^2) = 9\pi \) square inches. The area of the missing sector is \( \frac{5}{6} \cdot 9\pi = 1.5\pi \) square inches. The area of the original circle without the missing sector is \( \frac{5}{6} \cdot 9\pi = 7.5\pi \) square inches.

   c. The surface area includes a circle with a radius of 2.5 inches and a sector that is \( \frac{5}{6} \) of a circle with a radius of 3 inches. The surface area is \( \pi (2.5)^2 + 7.5\pi = 13.75\pi \) square inches.

2. The volume of a cone is \( \frac{1}{3} \cdot \text{Area Of Circle} \cdot \text{Height}, \)
   \( V = \frac{1}{3} \pi r^2 h. \)

3. The surface area of a cone is area of the base plus the product of \( \pi \), the radius, and the slant height, or \( S = \pi r^2 + \pi rl \). The volume is \( \frac{1}{3} \) times the area of the base times the height of the cone, or \( V = \frac{1}{3} \pi r^2 h. \)

4. Sample answer:

<table>
<thead>
<tr>
<th>Cone Volume and Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remaining arc without segment</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>5( \pi )</td>
</tr>
<tr>
<td>4( \pi )</td>
</tr>
<tr>
<td>3( \pi )</td>
</tr>
<tr>
<td>2( \pi )</td>
</tr>
</tbody>
</table>

Each time a \( 60^\circ \) sector is removed, the radius decreases by \( \frac{1}{6} \) of the original radius and the height increases.

The surface area decreases.

11.7 Monitoring Progress (pp. 642–644)

1. \( S = \pi r^2 + \pi \ell \)
   \( = \pi \cdot 7.8^2 + \pi \cdot 7.8 \cdot 10 \)
   \( \approx 436.18 \)

   The surface area is about 436.18 square meters.

2. \( V = \frac{1}{3} \pi r^2 h \)
   \( = \frac{1}{3} \cdot \pi \cdot 7^2 \cdot 13 \)
   \( = 212.33\pi \)
   \( \approx 667.06 \)

   The volume is about 667.06 cubic inches.

3. \( 8^2 = h^2 + 5^2 \)
   \( h^2 = 64 - 25 \)
   \( h = \sqrt{39} \)

   \( V = \frac{1}{3} \pi r^2 h \)
   \( = \frac{1}{3} \cdot \pi \cdot 5^2 \cdot \sqrt{39} \)
   \( = 25\sqrt{39} \pi \)
   \( \approx 163.49 \)

   The volume is about 163.49 cubic meters.

4. The scale factor is \( K = \frac{\text{Height of cone } D}{\text{Height of cone } C} = \frac{2}{8} = \frac{1}{4} \).

   Volume of cone \( D = \frac{K^3}{384\pi} \)
   Volume of cone \( C = \left( \frac{1}{4} \right)^3 \)
   Volume of cone \( D = 6\pi \)

   The volume of cone \( D \) is 6\( \pi \) cubic centimeters.
5. The volume of the cylinder is \((3^2 \cdot \pi) \cdot 10 \approx 282.74\).
   The volume of the cone is \(\frac{1}{3}(3^2 \cdot \pi) \cdot 5 \approx 47.12\).
   Volume of composite solid = cylinder + cone
   \(\approx 282.74 + 47.12 = 329.86\)
   The volume of the composite solid is about 329.86 cubic centimeters.

11.7 Exercises (pp. 645–646)

Vocabulary and Core Concept Check

1. Sample answer: Pyramids have a polygonal base and cones have a circular base. They both have sides that meet at a single vertex.

2. The volume of a cone with radius \(r\) and height \(h\) is \(\frac{1}{3}\) of the volume of a cylinder with radius \(r\) and height \(h\).

Monitoring Progress and Modeling with Mathematics

3. \(S = \pi r^2 + \pi r \ell\)
   \(= \pi \cdot 8^2 + \pi \cdot 8 \cdot 16\)
   \(= 64\pi + 128\pi\)
   \(= 192\pi \approx 603.19\)
   The surface area is about 603.19 square inches.

4. \(S = \pi r^2 + \pi \ell \ell\)
   \(= \pi \cdot 5.5^2 + \pi \cdot 5.5 \cdot 7.2\)
   \(= 30.25\pi + 39.6\pi\)
   \(= 69.85\pi \approx 219.44\)
   The surface area is about 219.44 square centimeters.

5. Find the slant height.
   \(\ell^2 = h^2 + r^2\)
   \(\ell^2 = 12^2 + 9^2\)
   \(\ell = \sqrt{225}\)
   \(\ell = 15\)
   \(S = \pi r^2 + \pi \ell \ell\)
   \(= \pi \cdot 9^2 + \pi \cdot 9 \cdot 15\)
   \(= 81\pi + 135\pi\)
   \(= 216\pi \approx 678.58\)
   The surface area is about 678.58 square inches.

6. The radius is \(11.2 \div 2 = 5.6\).
   Find the slant height.
   \(\ell^2 = h^2 + r^2\)
   \(\ell^2 = 9.2^2 + 5.6^2\)
   \(\ell = \sqrt{116}\)
   \(S = \pi r^2 + \pi \ell\ell\)
   \(= \pi \cdot 5.6^2 + \pi \cdot 5.6 \cdot \sqrt{116}\)
   \(= 31.36\pi + 39.6\pi\)
   \(= 219.44\)
   The surface area is about 219.44 square feet.

7. \(V = \frac{1}{3} \pi r^2 h\)
   \(= \frac{1}{3} \cdot \pi \cdot 10^2 \cdot 13\)
   \(= \frac{1300}{3} \cdot \pi\)
   \(= 433.33\pi \approx 1361.36\)
   The volume is about 1361.36 cubic millimeters.

8. \(V = \frac{1}{3} \pi r^2 h\)
   \(= \frac{1}{3} \cdot \pi \cdot l^2 \cdot 2\)
   \(= \frac{2}{3} \pi \approx 2.09\)
   The volume is about 2.09 cubic meters.

9. The radius is \(11.5 \div 2 = 5.75\).
   \(V = \frac{1}{3} \pi r^2 h\)
   \(= \frac{1}{3} \cdot \pi \cdot 5.75^2 \cdot 15.2\)
   \(= 167.52\pi \approx 526.27\)
   The volume is about 526.27 cubic inches.

10. Find the height.
    \(\ell^2 = h^2 + r^2\)
    \(6^2 = h^2 + 3^2\)
    \(h^2 = 36 - 9\)
    \(h^2 = 27\)
    \(h = \sqrt{27} = 3\sqrt{3}\)
    \(V = \frac{1}{3} \pi r^2 h\)
    \(= \frac{1}{3} \cdot \pi \cdot 3^2 \cdot 3\sqrt{3}\)
    \(= 9\pi\sqrt{3} \approx 48.97\)
    The volume is about 48.97 cubic feet.
Chapter 11

11. \( S = \pi r^2 + \pi r \ell \)

\[
75.4 = \pi \cdot 3^2 + \pi \cdot 3 \cdot \ell
\]

\[
\frac{47.13}{3\pi} = \ell
\]

The slant height \( \ell \) is about 5 centimeters.

\[ h^2 + r^2 = \ell^2 \]

\[ h^2 + 3^2 = 5^2 \]

\[ h^2 + 9 = 25 \]

\[ h^2 = 16 \]

\[ h = 4 \]

The height is about 4 centimeters.

12. \( V = \frac{1}{3} \pi r^2 h \)

\[
216\pi = \frac{1}{3} \pi \cdot r^2 \cdot 18
\]

\[
\frac{216\pi}{6\pi} = r^2
\]

\[ \sqrt{36} = r \]

\[ 6 = r \]

The radius is 6 inches.

13. The scale factor is \( K = \frac{\text{Radius of cone B}}{\text{Radius of cone A}} = \frac{8}{4} = 2. \)

Volume of cone B

Volume of cone A

\[
\frac{\text{Volume of cone B}}{32\pi} = (2)^3
\]

Volume of cone B = 256\pi

The volume of cone B is 256\pi cubic feet.

14. The scale factor is \( K = \frac{\text{Height of cone B}}{\text{Height of cone A}} = \frac{4}{10} = \frac{2}{5} \)

Volume of cone B

Volume of cone A

\[
\frac{\text{Volume of cone B}}{120\pi} = \left(\frac{2}{5}\right)^3
\]

Volume of cone B = 7.68\pi

The volume of cone B is 7.68\pi cubic meters.

15. The volume of the cylinder is \((3^2 \cdot \pi) \cdot 7 \approx 197.92.\)

The volume of the cone is \(\frac{1}{3}(3^2 \cdot \pi) \cdot 3 \approx 28.27.\)

Volume of composite solid = cylinder + cone

\[ \approx 197.92 + 28.27 = 226.19 \]

The volume of the composite solid is about 226.19 cubic centimeters.

16. The volume of the prism is \(5.1 \cdot 5.1 \cdot 5.1 = 132.65.\)

The volume of the cone is \(\frac{1}{3}(2.55^2 \cdot \pi) \cdot 5.1 \approx 34.73.\)

Volume of composite solid = prism + cone

\[ \approx 132.65 + 34.73 = 97.92 \]

The volume of the composite solid is about 97.92 cubic meters.

17. To double the volume, multiply the height by 2.

\[ V = \frac{1}{3} \pi r^2 h \]

\[ 2V = \frac{1}{3} \pi r^2 \cdot 2h \]

To double the volume, multiply the radius by \(\sqrt{2}.\)

\[ V = \frac{1}{3} \pi r^2 h \]

\[ 2V = \frac{1}{3} \pi (\sqrt{2}r)^2 h \]

The original volume is \( V = \frac{1}{3} \pi r^2 h \) and the new volume is \( V = \frac{2}{3} \pi r^2 h.\)

18. a. The volume of the cone-shaped container is \(\frac{1}{3}\) the volume of the cylindrical container, with both having the same radius and height. So, 3 of the smaller bags of popcorn will equal the larger bag of popcorn.

b. The cylindrical container of popcorn will give more for your money. Since 3 of the smaller containers equals the larger container, 3 of the smaller containers cost $3.75, and the larger container will cost $2.50.

19. The radius is the shorter leg of a \(30^\circ-60^\circ-90^\circ\) triangle.

\[ \frac{22}{\sqrt{3}} = \text{shorter leg} \cdot \sqrt{3} \]

\[ \frac{22}{\sqrt{3}} = \text{shorter leg} \]

\[ V = \frac{1}{3} \pi r^2 h \]

\[ \approx \frac{1}{3} \pi \left(\frac{22}{\sqrt{3}}\right)^2 22 \]

\[ \approx \frac{1}{3} \pi \left(\frac{484}{3}\right) 22 \]

\[ \approx 3716.85 \]

The volume of the cone is about 3716.85 cubic feet.

20. The radius is 7 yards. Find the height.

\[ \tan 32^\circ = \frac{r}{h} \]

\[ h = \frac{7}{\tan 32^\circ} \approx 11.2 \]

\[ V = \frac{1}{3} \pi r^2 h \]

\[ = \frac{1}{3} \pi \cdot 7^2 \cdot 11.2 \]

\[ \approx 574.70 \]

The volume of the cone is about 574.70 cubic yards.
Chapter 11

21. The volume of the cylinder is \((2.5^2 \cdot \pi) \cdot 7.5 \approx 147.26\).

The volume of the cone is \(\frac{1}{3}(2.5^2 \cdot \pi) \cdot 4 \approx 26.18\).

Volume of composite solid = cylinder + cone
\(\approx 147.26 + 26.18 = 173.44\)

The volume of the feeder is about 173.44 cubic inches.
The cat eats 1 cup per day. So, for 10 days, the cat eats
10 \(\cdot 14.4 = 144\) cubic inches. The feeder holds about
173.44 cubic inches, so there is enough food for 10 days.

22. \(V = \frac{1}{3} \pi r^2 h\)
\(= \frac{1}{3} \cdot \pi \cdot 5^2 \cdot 10\)
\(\approx 261.80\) cm\(^3\)

Retention rate = 80 - 65
= 15 milliliters per second
\(\frac{261.80}{15} = 17.453\) seconds

So, it will be about 17.45 seconds before the funnel overflows.

23. The outside surface area of the cup equals half the surface area of the original circle.
\(m \angle ABC = 60^\circ\)

24. Let \(x\) be the height of one of the smaller cones, and greater than 0 and less than \(h\).

\[V_{large} = V_1 + V_2\]

\[V_1 = \frac{1}{3} \pi r_1^2 h\]

\[V_2 = \frac{1}{3} \pi r_2^2 (h - x)\]

So, your friend is correct.

25. Let \(x\) be the height of one of the smaller cones, and greater than 0 and less than \(h\).

\[V_{large} = V_1 + V_2\]

\[V_1 = \frac{1}{3} \pi r_1^2 h\]

\[V_2 = \frac{1}{3} \pi r_2^2 (h - x)\]

So, your friend is correct.

26. Cone 1: \(V = \frac{1}{3} \cdot \pi \cdot 15^2 \cdot 20\)
\(V = 1500\pi\) cubic units

Cone 2: \(V = \frac{1}{3} \cdot \pi \cdot 20^2 \cdot 15\)
\(V = 2000\pi\) cubic units

Cone 3:
To find the radius, use the Geometric Leg Mean Theorem.
\(20^2 = a \cdot 25\)
\[a = \frac{400}{25} = 16\]
\(20^2 = r^2 + 16^2\)
\(400 = r^2 + 256\)
\(r^2 = 144\)
\(r = 12\)

\[V = \frac{1}{3} \cdot \pi \cdot 12^2 \cdot 25\]
\(V = 1200\pi\) cubic units
Chapter 11

Maintaining Mathematical Proficiency

27. \( A = \pi r^2 \)
   
   \[ = \pi \cdot 7^2 \]
   \[ = 49\pi \approx 153.94 \]
   The area of the circle is about 153.94 square feet.

28. The radius is \( 22 \div 2 = 11 \) centimeters.
   
   \( A = \pi r^2 \)
   
   \[ = \pi \cdot 11^2 \]
   \[ = 11\pi \approx 380.13 \]
   The area of the circle is about 380.13 square centimeters.

29. \( A = \pi r^2 \)
   
   \[ = \frac{256\pi}{\pi} \]
   \[ = 256 \]
   \[ r = \sqrt{256} = 16 \]
   The diameter is \( 2 \cdot 16 = 32 \) units.

30. \( A = \pi r^2 \)
   
   \[ = \frac{529\pi}{\pi} \]
   \[ = 529 \]
   \[ r = \sqrt{529} = 23 \]
   The radius is 23 units.

11.8 Explorations (p. 647)

1. The surface area of a sphere is \( S = 4\pi r^2 \).

2. Volume_{cylinder} = \text{Area of base} \cdot \text{Height}
   
   \[ = (\pi r^2) \cdot 2r \]
   \[ = 2\pi r^3 \]
   
   \[ 3 \cdot \left( \frac{1}{2} \text{Volume}_{sphere} \right) = 2\pi r^3 \]
   \[ \text{Volume}_{sphere} = 2\pi r^3 \cdot \frac{2}{3} \]
   \[ = \frac{4}{3}\pi r^3 \]

3. To find the surface area of a sphere, use the formula \( S = 4\pi r^2 \). To find the volume of a sphere, use the formula \( V = \frac{4}{3}\pi r^3 \).

4. a. Surface area: \( S = 4\pi(3)^2 = 36\pi \approx 113.10 \text{ in.}^2 \)
   
   Volume: \( V = \frac{4}{3}\pi(3)^3 = 36\pi \approx 113.10 \text{ in.}^3 \)

   b. Surface area: \( S = 4\pi(2)^2 = 16\pi \approx 50.27 \text{ cm}^2 \)
   
   Volume: \( V = \frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \approx 33.51 \text{ cm}^3 \)

5. The radius is \( 36 \div 2 = 18 \) inches.
   
   \[ V = \frac{4}{3}\pi r^3 \]
   \[ = \frac{4}{3}\pi(18)^3 \]
   \[ = 7776\pi \]
   \[ \approx 24,429.02 \]
   The volume of the sphere is about 24,429.02 cubic inches.

6. \( S = 4\pi r^2 \)
   
   \[ 576\pi = 4\pi r^2 \]
   \[ r^2 = 144 \]
   \[ r = \sqrt{144} = 12 \]
   
   \[ V = \frac{4}{3}\pi r^3 \]
   \[ = \frac{4}{3}\pi(12)^3 \]
   \[ = 2304\pi \]
   \[ \approx 7238.23 \]
   The volume of the sphere is about 7238.23 cubic centimeters.

11.8 Monitoring Progress (pp. 648–651)

1. The radius is \( 40 \div 2 = 20 \) feet.
   
   \[ S = 4\pi r^2 \]
   \[ = 4\pi(20)^2 \]
   \[ = 1600\pi \approx 5026.55 \]
   The surface area is about 5026.55 square feet.

2. \( C = 6\pi \)
   
   \[ 2\pi \cdot r = 6\pi \]
   \[ r = 3 \]
   
   \[ S = 4\pi r^2 \]
   \[ = 4\pi(3)^2 \]
   \[ = 36\pi \approx 113.10 \]
   The surface area is about 113.10 square feet.

3. \( S = 4\pi r^2 \)
   
   \[ 30\pi = 4\pi r^2 \]
   \[ r^2 = \frac{30}{4} \]
   \[ r = \sqrt{\frac{15}{2}} = 2.74 \]
   The radius of the sphere is about 2.74 meters.

4. \( V = \frac{4}{3}\pi r^3 \)
   
   \[ = \frac{4}{3}\pi(5)^3 \]
   \[ = \frac{500}{3}\pi \]
   \[ \approx 523.60 \]
   The volume of the sphere is about 523.60 cubic yards.

5. The radius is \( 36 \div 2 = 18 \) inches.
   
   \[ V = \frac{4}{3}\pi r^3 \]
   \[ = \frac{4}{3}\pi(18)^3 \]
   \[ = 7776\pi \]
   \[ \approx 24,429.02 \]
   The volume of the sphere is about 24,429.02 cubic inches.
Chapter 11

7. Volume of the hemisphere:
   \[ V = \frac{1}{2} \left( \frac{4}{3} \pi \cdot 1^3 \right) = \frac{2}{3} \pi \]
   Volume of the cone:
   \[ V = \frac{1}{3} \pi \cdot 1^3 \cdot 5 = \frac{5}{3} \pi \]
   Volume of the hemisphere + Volume of the cone
   \[ = \frac{2}{3} \pi + \frac{5}{3} \pi \]
   \[ = \frac{7}{3} \pi \]
   The volume of the composite solid is \( \frac{7}{3} \pi \) or about 7.33 cubic meters.

8. \[ S = 4\pi r^2 \]
   \[ 1024\pi = 4\pi r^2 \]
   \[ r^2 = 256 \]
   \[ r = \sqrt{256} \]
   \( r = 16 \)
   The radius of the sphere is 16 inches.

9. \[ S = 4\pi r^2 \]
   \[ 900\pi = 4\pi r^2 \]
   \[ r^2 = 225 \]
   \[ r = \sqrt{225} \]
   \[ r = 15 \]
   \[ d = 2r = 30 \]
   The diameter of the sphere is 30 meters.

11.8 Exercises (pp. 652–654)

Vocabulary and Core Concept Check
1. The plane must go through the center of the sphere in order to form a great circle or equator.

2. Sample answer: A hemisphere is one-half of a sphere.

Monitoring Progress and Modeling with Mathematics
3. \[ S = 4\pi r^2 \]
   \[ = 4\pi(4)^2 \]
   \[ = 64\pi \approx 201.06 \]
   The surface area is about 201.06 square feet.

4. \[ S = 4\pi r^2 \]
   \[ = 4\pi(7.5)^2 \]
   \[ = 225\pi \approx 706.86 \]
   The surface area is about 706.86 square centimeters.

5. The radius is 18.3 \div 2 = 9.15 feet.
   \[ S = 4\pi r^2 \]
   \[ = 4\pi(9.15)^2 \]
   \[ = 334.89\pi \approx 1052.09 \]
   The surface area is about 1052.09 square meters.

6. \[ C = 4\pi \]
   \[ 2\pi r = 4\pi \]
   \[ r = 2 \]
   So, the radius is 2 feet.
   \[ S = 4\pi r^2 \]
   \[ = 4\pi(2)^2 \]
   \[ = 16\pi \approx 50.27 \]
   The surface area is about 50.27 square feet.

7. \[ S = 4\pi r^2 \]
   \[ 4\pi = 4\pi r^2 \]
   \[ r^2 = 1 \]
   \[ r = 1 \]
   The radius of the sphere is 1 foot.

8. \[ S = 4\pi r^2 \]
   \[ 1024\pi = 4\pi r^2 \]
   \[ r^2 = 256 \]
   \[ r = \sqrt{256} \]
   \[ r = 16 \]
   The radius of the sphere is 16 inches.

9. \[ S = 4\pi r^2 \]
   \[ 900\pi = 4\pi r^2 \]
   \[ r^2 = 225 \]
   \[ r = \sqrt{225} \]
   \[ r = 15 \]
   \[ d = 2r = 30 \]
   The diameter of the sphere is 30 meters.

10. \[ S = 4\pi r^2 \]
    \[ 196\pi = 4\pi r^2 \]
    \[ r^2 = 49 \]
    \[ r = \sqrt{49} \]
    \[ r = 7 \]
    \[ d = 2r = 14 \]
    The diameter of the sphere is 14 centimeters.

11. \[ S = \frac{1}{2} \cdot 4\pi r^2 \]
    \[ = 2\pi r^2 \]
    \[ = 2\pi(5)^2 \]
    \[ = 50\pi \approx 157.08 \]
    The surface area of the hemisphere is about 157.08 square meters.

12. The radius is 12 \div 2 = 6 inches.
    \[ S = \frac{1}{2} \cdot 4\pi r^2 \]
    \[ = 2\pi r^2 \]
    \[ = 2\pi(6)^2 \]
    \[ = 72\pi \approx 226.19 \]
    The surface area of the hemisphere is about 226.19 square inches.

13. \[ V = \frac{4}{3} \pi r^3 \]
    \[ = \frac{4}{3} \pi(8)^3 \]
    \[ = \frac{2048}{3} \pi \approx 2144.66 \]
    The volume of the sphere is about 2144.66 cubic meters.

14. \[ V = \frac{4}{3} \pi r^3 \]
    \[ = \frac{4}{3} \pi(4)^3 \]
    \[ = \frac{256}{3} \pi \approx 268.08 \]
    The volume of the sphere is about 268.08 cubic feet.
Chapter 11

15. The radius is $22 \div 2 = 11$ yards.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (11)^3$

$= \frac{5324}{3}\pi \approx 5575.28$

The volume of the sphere is about 5575.28 cubic yards.

16. The radius is $14 \div 2 = 7$ feet.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (7)^3$

$= \frac{1372}{3}\pi \approx 1436.76$

The volume of the sphere is about 1436.76 cubic feet.

17. $C = 20\pi$

$2\pi r = 20\pi$

$r = 10$

The radius is 10 centimeters.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (10)^3$

$= \frac{4000}{3}\pi \approx 4188.79$

The volume of the sphere is about 4188.79 cubic centimeters.

18. $C = 7\pi$

$2\pi r = 7\pi$

$r = 3.5$

The radius is 3.5 inches.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (3.5)^3$

$= 57.17\pi \approx 179.59$

The volume of the sphere is about 179.59 cubic inches.

19. $S = 4\pi r^2$

$16\pi = 4\pi r^2$

$r = \sqrt{4}$

$r = 2$

The radius is 2 feet.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (2)^3$

$= \frac{32}{3}\pi \approx 33.51$

The volume of the sphere is about 33.51 cubic feet.

20. $S = 4\pi r^2$

$484\pi = 4\pi r^2$

$r = \sqrt{121}$

$r = 11$

The radius is 11 centimeters.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (11)^3$

$= \frac{5324}{3}\pi \approx 5575.28$

The volume of the sphere is about 5575.28 cubic centimeters.

21. The radius was squared instead of cubed.

$V = \frac{4}{3}\pi (6)^3$

$= 288\pi$

$\approx 904.78 \text{ ft}^3$

22. The diameter was used instead of the radius.

$V = \frac{4}{3}\pi (1.5)^3$

$= 4.50\pi$

$\approx 14.14 \text{ in.}^3$

23. Volume of cylinder:

$V = \pi r^2h$

$= \pi (5)^29$

$= 225\pi$

Volume of hemisphere:

$V = \frac{1}{2} \left( \frac{4}{3}\pi r^3 \right)$

$= \frac{2}{3}\pi (5)^3$

$= \frac{250}{3}\pi$

Volume of cylinder $- \text{ Volume of hemisphere}$

$= \frac{225\pi}{3} - \frac{250}{3}\pi$

$= \frac{675\pi}{3} - \frac{250\pi}{3}$

$= \frac{425}{3}\pi \approx 445.06$

The volume of the composite solid is about 445.06 cubic inches.
24. Volume of cone:
\[ V = \frac{1}{3} \pi r^2 h \]
\[ = \frac{1}{3} \pi \cdot 6^2 \cdot 12 \]
\[ = 144\pi \]
Volume of hemisphere:
\[ V = \frac{1}{2} \cdot \frac{2}{3} \pi r^3 \]
\[ = \frac{2}{3} \pi (6)^3 \]
\[ = 144\pi \]
Volume of cone + Volume of hemisphere
\[ = 144 + 144\pi \]
\[ = 288\pi \approx 904.78 \]
The volume of the composite solid is about 904.78 cubic feet.

25. Volume of cylinder:
\[ V = \pi r^2 h = \pi (10)^2 \cdot 18 = 1800\pi \]
Volume of hemisphere:
\[ V = \frac{1}{2} \cdot \frac{2}{3} \pi r^3 \]
\[ = \frac{2}{3} \pi (10)^3 \]
\[ = \frac{2000}{3}\pi \]
Volume of cylinder − Volume of hemisphere
\[ = 1800\pi + \frac{2000}{3}\pi \]
\[ = \frac{5400}{3}\pi + \frac{2000}{3}\pi \]
\[ = \frac{7400}{3}\pi \approx 7749.26 \]
The volume of the composite solid is about 7749.26 cubic centimeters.

26. Volume of cylinder:
\[ V = \pi r^2 h \]
\[ = \pi (6)^2 \cdot 14 \]
\[ = 504\pi \]
Volume of one hemisphere:
\[ V = \frac{1}{2} \cdot \frac{2}{3} \pi r^3 \]
\[ = \frac{2}{3} \pi (6)^3 \]
\[ = 144\pi \]
Volume of cylinder + 2 • Volume of hemisphere
\[ = 504\pi + 2 \cdot 144\pi \]
\[ = 792\pi \approx 2488.14 \]
The volume of the composite solid is about 2488.14 cubic meters.

27. The radius is 8.5 ÷ 2 = 4.25 inches
\[ S = 4\pi r^2 \]
\[ = 4\pi(4.25)^2 \]
\[ = 72.25\pi \]
\[ \approx 226.98 \]
\[ V = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} \pi (4.25)^3 \]
\[ = 102.35\pi \]
\[ \approx 321.56 \]
The surface area of the bowling ball is about 226.98 square inches and the volume is about 321.56 cubic inches.

28. \[ C = 29.5 \]
\[ 2\pi r = 29.5 \]
\[ r = 4.7 \]
The radius is about 4.7 inches.
\[ S = 4\pi r^2 \]
\[ = 4\pi(4.7)^2 \]
\[ = 277.59 \]
\[ V = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} \pi (4.7)^3 \]
\[ = 434.89 \]
The surface area of the basketball is about 277.59 square inches and the volume is about 434.89 cubic inches.

29. \[ C = 12 \]
\[ 2\pi r = 12 \]
\[ r = 1.9 \]
The radius is about 1.9 inches.
\[ S = 4\pi r^2 \]
\[ = 4\pi(1.9)^2 \]
\[ = 45.84 \]
\[ V = \frac{4}{3} \pi r^3 \]
\[ = \frac{4}{3} \pi (1.9)^3 \]
\[ = 28.73 \]
The surface area of the softball is about 45.84 square inches and the volume is about 28.73 cubic inches.
30. The radius is $1.7 \div 2 = 0.85$ inch.

\[ S = 4\pi r^2 \]
\[ = 4\pi(0.85)^2 \]
\[ \approx 9.08 \]
\[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(0.85)^3 \]
\[ \approx 2.57 \]

The surface area of the golf ball is about 9.08 square inches and the volume is about 2.57 cubic inches.

31. \[ C = 26 \]
\[ 2\pi r = 26 \]
\[ r = 4.14 \]

The radius is about 4.14 inches.

\[ S = 4\pi r^2 \]
\[ = 4\pi(4.14)^2 \]
\[ \approx 215.18 \]
\[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(4.14)^3 \]
\[ \approx 297.23 \]

The surface area of the volleyball is about 215.18 square inches and the volume is about 297.23 cubic inches.

32. \[ C = 9 \]
\[ 2\pi r = 9 \]
\[ r \approx 1.43 \]

The radius is about 1.43 inches.

\[ S = 4\pi r^2 \]
\[ = 4\pi(1.43)^2 \]
\[ \approx 25.70 \]
\[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(1.43)^3 \]
\[ \approx 12.25 \]

The surface area of the baseball is about 25.70 square inches and the volume is about 12.25 cubic inches.

33. no; If the radius doubles, the surface area will be multiplied by 4. Let \( r = 2r \), then the surface area is \( 4\pi(2r)^2 \), which is \( 4\pi \times 4r^2 = 4(4\pi r^2) \), and is 4 times the original surface area.

34. The radius is $18 \div 2 = 9$ inches.

\[ S = 4\pi r^2 \]
\[ = 4\pi(9)^2 \]
\[ = 324\pi \]
\[ \approx 1017.88 \]
\[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(9)^3 \]
\[ = 972\pi \]
\[ \approx 3053.63 \]

The solid formed is a sphere and the surface area is about 1017.88 square inches, and the volume is about 3053.63 cubic inches.

35. Volume of cylindrical portion:

\[ V = \pi r^2 h \]
\[ = \pi(10)^2 \cdot 60 \]
\[ = 6000\pi \]

Volume of top hemisphere:

\[ V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \]
\[ = \frac{2}{3}\pi(10)^3 \]
\[ = \frac{2000}{3}\pi \]

Volume of cylindrical portion + Volume of top hemisphere

\[ = 6000\pi + \frac{2000}{3}\pi = \frac{18,000}{3}\pi + \frac{2000}{3}\pi = \frac{20,000}{3}\pi \]
\[ \approx 20,943.95 \]

The volume of the silo is about 20,943.95 cubic feet.

36. \[ C = 8 \]
\[ 2\pi r = 8 \]
\[ r \approx 1.27 \]

The radius of the tennis ball is about 1.27 inches.

a. \[ V = \frac{4}{3}\pi r^3 \]
\[ = \frac{4}{3}\pi(1.27)^3 \]
\[ \approx 8.58 \]

The volume of the tennis ball is about 8.58 cubic inches.

b. \[ V = \pi r^2 h \]
\[ = \pi(1.43)^2 \cdot 8 \]
\[ \approx 51.39 \]

Volume of cylinder – Volume of 3 tennis balls
\[ \approx 51.39 - 3 \cdot 8.58 \]
\[ = 25.65 \]

The amount of space not taken up in the cylinder by the tennis balls is about 25.65 cubic inches.
Chapter 11

37. Analyzing data:
   
   a. Worked-Out Solutions
   
<table>
<thead>
<tr>
<th>Radius</th>
<th>Surface area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 in.</td>
<td>$4\pi(3)^2 = 36\pi$ in.$^2$</td>
<td>$\frac{4}{3}\pi(3)^3 = 36\pi$ in.$^3$</td>
</tr>
<tr>
<td>6 in.</td>
<td>$4\pi(6)^2 = 144\pi$ in.$^2$</td>
<td>$\frac{4}{3}\pi(6)^3 = 288\pi$ in.$^3$</td>
</tr>
<tr>
<td>9 in.</td>
<td>$4\pi(9)^2 = 324\pi$ in.$^2$</td>
<td>$\frac{4}{3}\pi(9)^3 = 972\pi$ in.$^3$</td>
</tr>
<tr>
<td>12 in.</td>
<td>$4\pi(12)^2 = 576\pi$ in.$^2$</td>
<td>$\frac{4}{3}\pi(12)^3 = 2304\pi$ in.$^3$</td>
</tr>
</tbody>
</table>

b. When the radius doubles, the surface area is multiplied by 4 or $2^2$. When the radius is tripled, the surface area is multiplied by 9 or $3^2$. When the radius is quadrupled, the surface area is multiplied by 16 or $4^2$.

c. When the radius doubles, the volume is multiplied by 8 or $2^3$. When the radius is tripled, the volume is multiplied by 27 or $3^3$. When the radius is quadrupled, the volume is multiplied by 64 or $4^3$.

38. The radius is $\frac{4(x + 3)}{2} = 2(x + 3)$.
   
   $S = 4\pi r^2$
   
   $784\pi = 4\pi(2(x + 3))^2$
   
   $784\pi = 4\pi \cdot 4 \cdot (x + 3)^2$
   
   $784\pi = 16\pi(x + 3)^2$
   
   $\frac{784\pi}{16\pi} = (x + 3)^2$
   
   $(x + 3)^2 = 49$
   
   $x + 3 = 7$
   
   $x = 4$

39. a. Surface area of Earth:
   
   $S = 4\pi r^2$
   
   $= 4\pi(3960)^2$
   
   $= 62,726,400\pi \approx 197,060,797$ mi$^2$
   
   Surface area of the moon:
   
   $S = 4\pi r^2$
   
   $= 4\pi(1080)^2$
   
   $= 4,665,600\pi \approx 14,657,415$ mi$^2$

b. Surface area of Earth
   
   Surface area of moon
   
   $\frac{62,726,400\pi}{4,665,600\pi} = \frac{121}{9} = \left(\frac{11}{3}\right)^2 \approx 13.4$

   The surface area of Earth is about 13.4 times greater than the surface area of the moon.

c. The amount of water on the surface of Earth is about $0.70 \cdot 62,726,400\pi \approx 137,942,558$ square miles.

40. a. $S = 2\pi rh$
   
   $= 2\pi(3960)(3250)$
   
   $= 25,740,000\pi$
   
   $= 80,864,594$

   The surface area of the Torrid Zone is about 80,864,594 square miles.

b. The probability that a meteorite is equally likely to hit in the Torrid Zone is about
   
   $\frac{25,740,000\pi}{62,726,400\pi} = \frac{325}{792} \approx 0.410, 41\%$.

41. The cube with a volume of 64 cubic inches has a side length of $\sqrt[3]{64} = 4$ inches. The sphere inside the cube has a radius of 2 inches. The surface area of the sphere is $4\pi(2)^2 = 16\pi$, or about 50.27 square inches.

42. Let $r = h$.

   Volume of hemisphere $= \frac{2}{3}\pi r^3$

   Volume of hemisphere $= \frac{2}{3}\pi h^3$

   Volume of cone $= \frac{1}{3}\pi r^2h$

   Volume of cone $= \frac{1}{3}\pi h^2 \cdot h$

   Volume of cone $= \frac{1}{3}\pi h^3$

   $2(\text{Volume of cone}) = \frac{2}{3}\pi h^3$

   Volume of hemisphere = $2(\text{Volume of cone})$

   The volume of the hemisphere is greater than the volume of the cone by a factor of 2.

43. $V = \frac{\frac{4}{3}\pi r^3}{(4\pi r^2)}$

   $V = \frac{r}{3}$

   $V = \frac{5r}{3}$

44. The area of a lune is the surface area of the sphere times the measure of the sector divided by 360°.

   $S = \frac{4\pi r^2 \cdot \theta}{360^\circ}$

   Convert 360° to radians.

   $360^\circ \cdot \frac{\pi}{180^\circ} = 2\pi$ radians

   So, $S = \frac{4\pi r^2 \theta}{2\pi} = 2r^2 \theta$. 

416 Geometry

worked-out solutions

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45. Volume$_{cylinder} = $Volume$_{sphere}$

\[
\pi r^2 h = \frac{4}{3} \pi r^3 \\
\pi r^2 h = \frac{4}{3} \pi \cdot 1^3 \\
\pi r^2 h = \frac{4}{3} \pi \\
r = \sqrt[3]{\frac{4}{3} h} \\
r = -\frac{2}{\sqrt{3} h} \\
r^2 = \frac{4}{3h} \\
h = \frac{4}{3r^2} \\
r^2 h = \frac{4}{3}
\]

Choose any $r$ and $h$, where $r^2 h = \frac{4}{3}$ is a solution.

Sample answers:
- $r = 2$, $h = \frac{1}{3}$
- $r = 1$, $h = \frac{4}{3}$
- $r = \frac{1}{3}$, $h = 12$

46. a. $r = h + x$

\[
r^2 = a^2 + x^2 \\
5^2 = a^2 + x^2 \\
x = \sqrt{25 - 16} \\
x = \sqrt{9} = 3 \\
h = 5 - 3 = 2 \\
V = \frac{\pi h}{6}(3a^2 + h^2) \\
= \frac{\pi \cdot 2}{6}(3 \cdot 4^2 + 2^2) \\
\approx 54.45 \text{ in.}^3
\]

b. $r = h + x$

\[
r^2 = a^2 + x^2 \\
34^2 = 30^2 + x^2 \\
x = \sqrt{1156 - 900} \\
x = \sqrt{256} = 16 \\
h = 34 - 16 = 18 \\
V = \frac{\pi h}{6}(3a^2 + h^2) \\
= \frac{\pi \cdot 18}{6}(3 \cdot 30^2 + 18^2) \\
\approx 28,500.53 \text{ cm}^3
\]

c. $r = h + x$

\[
13 = 8 + x \\
x = 5 \\
r^2 = a^2 + x^2 \\
13^2 = a^2 + 5^2 \\
a = \sqrt{169 - 25} \\
a = \sqrt{144} = 12 \\
V = \frac{\pi h}{6}(3a^2 + h^2) \\
= \frac{\pi \cdot 8}{6}(3 \cdot 12^2 + 8^2) \\
= 2077.64 \text{ m}^3
\]

d. $r = h + x$

\[
75 = 54 + x \\
x = 21 \\
r^2 = a^2 + x^2 \\
75^2 = a^2 + 21^2 \\
a = \sqrt{5625 - 441} \\
a = \sqrt{5184} = 72 \\
V = \frac{\pi h}{6}(3a^2 + h^2) \\
= \frac{\pi \cdot 54}{6}(3 \cdot 72^2 + 54^2) \\
= 522,170.4 \text{ in.}^3
\]
47. Consider a vertical cross section through the cone vertex and sphere center. The cone maps to a triangle and the sphere maps to a circle. Where the triangle’s sides are tangent to the circle, the radii intersecting the sides are perpendicular to the tangent sides. The angle formed by each intersecting radius and tangent side is 90°. The right triangle formed by the vertex point of the cone, the center of the sphere, and the tangent point is a 30°-60°-90° triangle. Because the hypotenuse is 4 and the circle’s radius is the shorter leg, 2, or half of the hypotenuse. The angle opposite the length of 2 is 30°. So, the longer leg is $2\sqrt{3}$.

Because there is an identical congruent triangle reflected on the vertical hypotenuse axis, the vertex angle of the large cross sectional triangle is 2 times 30°, or 60°. So, the congruent base angles are $180° - 60° = 60°$ and the triangle formed by the upper vertex of the large triangle and the triangle base points is an equilateral triangle. By bisecting the upper vertex angle, two 30°-60°-90° triangles are formed with sides $4\sqrt{3}$, bases 2, and heights 6. From the triangle and circle, the cone radius is $2\sqrt{3}$ and the sphere radius was given as 2. The volume of the cone and sphere are as follows:

$$\text{Volume}_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \pi (2\sqrt{3})^2 \cdot 6$$

$$= \frac{1}{3} \pi \cdot 4 \cdot 3 \cdot 6 = 24\pi \approx 75.4$$

Slant height = $4\sqrt{3}$

Surface Area$_{\text{cone}} = \pi r^2 + \pi rl$

$$= \pi (2\sqrt{3})^2 + \pi 2\sqrt{3} \cdot 4\sqrt{3}$$

$$= 12\pi + 24\pi = 36\pi \approx 113.1$$

The volume of the cone is about 75.40 cubic inches. The surface area of the cone is about 113.1 square inches.

**Maintaining Mathematical Proficiency**

48. $m\angle B = 180° - (26° + 35°)$

$$= 119°$$

$$\sin A \over a = \sin B \over b$$

$$\sin 26° \over a = \sin 119° \over 13$$

$$13 \sin 26° \over \sin 119° = a$$

$$a \approx 6.5$$

$$\sin B \over b = \sin C \over c$$

$$\sin 119° \over 13 = \sin 35° \over c$$

$$c = \frac{13 \sin 35°}{\sin 119°}$$

$$c \approx 8.5$$

In $\triangle ABC$, $m\angle B = 119°$, $a \approx 6.5$, and $c \approx 8.5$.

49. $m\angle A = 180° - (43° + 102°)$

$$= 35°$$

$$\sin A \over a = \sin B \over b$$

$$\sin 35° \over a = \sin 102° \over 21$$

$$21 \sin 35° \over \sin 102° = a$$

$$a \approx 12.3$$

$$\sin B \over b = \sin C \over c$$

$$\sin 102° \over 21 = \sin 43° \over c$$

$$c = \frac{21 \sin 43°}{\sin 102°}$$

$$c \approx 14.6$$

In $\triangle ABC$, $m\angle A = 35°$, $a \approx 12.3$, and $c \approx 14.6$. 
Chapter 11

50. \(a^2 = b^2 + c^2 - 2bc \cos A\)
\[23^2 = 24^2 + 20^2 - 2(24)(20) \cos A\]
\[529 = 576 + 400 - 960 \cos A\]
\[529 = 976 - 960 \cos A\]
\[-447 = -960 \cos A\]
\[-447 = \cos A\]
\[\cos A = \frac{-447}{960}\]
\[m\angle A = \cos^{-1}\left(\frac{-447}{960}\right) \approx 62.2^\circ\]
\[\sin A = \frac{\sin B}{b}\]
\[\sin 62.2^\circ = \frac{\sin B}{23}\]
\[\sin 62.2^\circ = \frac{\sin B}{24}\]
\[\sin B = \frac{24 \sin 62.2^\circ}{23}\]
\[m\angle B = \sin^{-1}\left(\frac{24 \sin 62.2^\circ}{23}\right) \approx 67.4^\circ\]
\[m\angle C = 180^\circ - (62.2^\circ + 67.4^\circ) = 50.4^\circ\]
In \(\triangle ABC\), \(m\angle A \approx 62.2^\circ\), \(m\angle B \approx 67.4^\circ\), and \(m\angle C \approx 50.4^\circ\).

51. \(a^2 = b^2 + c^2 - 2ab \cos A\)
\[a^2 = 15^2 + 24^2 - 2(15)(24) \cos 103^\circ\]
\[a^2 = 225 + 576 - 720 \cos 103^\circ\]
\[a^2 = 801 - 720 \cos 103^\circ\]
\[a = \sqrt{963} \approx 31.03\]
\[\sin A = \frac{\sin B}{b}\]
\[\sin 103^\circ = \frac{\sin B}{15}\]
\[\sin B = \frac{15 \sin 103^\circ}{31.03}\]
\[m\angle B = \sin^{-1}\left(\frac{15 \sin 103^\circ}{31.03}\right) \approx 28.1^\circ\]
\[m\angle C = 180^\circ - (103^\circ + 28.1^\circ) = 48.9^\circ\]
In \(\triangle ABC\), \(m\angle B \approx 28.1^\circ\), \(m\angle C \approx 48.9^\circ\), and \(a \approx 31.03\).

11.5–11.8 What Did You Learn? (p. 655)

1. Sample answer: yes; The larger container usually has a lesser unit cost.

2. Sample answer: The scale factor is \(\frac{1}{4}\) and the ratio of the volumes is the scale factor cubed.

3. Sample answer: Substitute \(r = 2x + 6\) into the surface area formula and set it equal to \(784\pi\), then solve for \(x\).

Chapter 11 Review (pp. 656–660)

1. \(C = \pi d\)
\[94.24 = \pi d\]
\[\frac{94.24}{\pi} = d\]
\[d \approx 30\] feet.
The diameter of circle \(P\) is about 30 feet.

2. Arc length of \(\overline{GH} = \frac{m\overarc{GH}}{360^\circ} \cdot C\)
\[5.5 = \frac{35^\circ}{360^\circ} \cdot C\]
\[\frac{5.5 \cdot 360^\circ}{35^\circ} = C\]
\[C \approx 56.57\] centimeters.
The circumference is about 56.57 centimeters.

3. Arc length of \(\overline{AB} = \frac{m\overarc{AB}}{360^\circ} \cdot C\)
\[115^\circ \cdot 2 \cdot 13 \cdot \pi \approx 26.09\] inches.
The arc length is about 26.09 inches.

4. \(C = \pi d = 26\pi\)

32 revolutions is about \(32 \cdot 26\pi \approx 2613.8\) inches.
\[2613.8 \approx 217.82\]
The tire travels about 218 feet in 32 revolutions.

5. Area of sector \(TWU = \frac{m\overarc{TWU}}{360^\circ} \cdot \text{Area of } \odot V\)
\[= \frac{240^\circ}{360^\circ} \cdot \pi \cdot 9^2\]
\[\approx 169.65\]
The area of the blue shaded region is about 169.65 square inches.

6. Area of rectangle = \(4 \cdot 6 = 24\)
Area of semicircle = \(\frac{1}{2} \cdot \pi \cdot 2^2 = 2\pi\)
Area of rectangle - Area of semicircle = \(24 - 2\pi \approx 17.72\)
The area of the blue shaded region is about 17.72 square inches.
7. Area of sector $RTQ = \frac{m\angle RQ}{360^\circ} \cdot \text{Area of } \odot S$ 
\[
\frac{27.93 \cdot 360^\circ}{50^\circ} = \text{Area of } \odot S
\]
\[
27.93 = \frac{\text{Area of pink sector}}{\text{Area of } \odot S}
\]
Area of circle $- \text{Area of pink sector} \approx 201.1 - 27.93 = 173.17$
The area of the shaded region is about 173.17 square feet.

8. $A = \frac{1}{2} d_1 d_2$
\[
= \frac{1}{2} \cdot 13 \cdot 20
\]
\[
= 130
\]
The area of the kite is 130 square units.

9. $A = \frac{1}{2} d_1 d_2$
\[
= \frac{1}{2} \cdot 12 \cdot 16
\]
\[
= 96
\]
The area of the kite is 96 square units.

10. $A = \frac{1}{2} d_1 d_2$
\[
= \frac{1}{2} \cdot 14 \cdot 15
\]
\[
= 105
\]
The area of the kite is 105 square units.

11. The central angle of the regular hexagon is $\frac{360^\circ}{6} = 60^\circ$.
The apothem bisects the central angle, which is $30^\circ$.
The side opposite the $30^\circ$ angle is 4.4 and the apothem (side opposite the $60^\circ$ angle) is $4.4\sqrt{3}$.
\[
A = \frac{1}{2} a \cdot ns
\]
\[
= \frac{1}{2} (4.4\sqrt{3})(6)(8.8)
\]
\[
= 201.20
\]
The area of the regular hexagon is about 201.2 square units.

12. To find the apothem, use the Pythagorean Theorem $c^2 = a^2 + b^2$.
\[
7.6^2 = a^2 + 2.6^2
\]
\[
a^2 = 57.76 - 6.76
\]
\[
a^2 = 51
\]
\[
a = 7.1
\]
\[
A \approx \frac{1}{2} a \cdot ns
\]
\[
= \frac{1}{2}(7.1)(9)(5.2)
\]
\[
\approx 166.14
\]
The area of the regular nonagon is about 166.14 square units.

13. To find the length of the side of the pentagon, use the Pythagorean Theorem $c^2 = a^2 + b^2$.
\[
4^2 = a^2 + 3.3^2
\]
\[
a^2 = 16 - 10.89
\]
\[
a^2 = 5.11
\]
\[
a \approx 2.26
\]
The side is $2 \cdot 2.26 = 4.52$.
\[
A = \frac{1}{2} a \cdot ns
\]
\[
= \frac{1}{2}(3.3)(5)(4.52)
\]
\[
\approx 37.29
\]
The area of the regular pentagon is about 37.29 square units.

14. The central angle of a regular octagon is $\frac{360^\circ}{8} = 45^\circ$.
The base angles of the isosceles triangle formed by the radii of the polygon measure $67.5^\circ$. The apothem is 6 inches.
\[
\tan 67.5^\circ = \frac{6}{x}
\]
\[
x = \frac{6}{\tan 67.5^\circ} \approx 2.49
\]
The length of one side of the octagon is $2 \cdot 2.49 = 4.98$ centimeters.
\[
A = \frac{1}{2} a \cdot ns
\]
\[
= \frac{1}{2}(6)(8)(4.98)
\]
\[
= 119.52
\]
The area of the platter is about 119.52 square inches.
15. The solid is a cone with height 9 and base radius 5.

![Cone Diagram]

16. The solid is a sphere with radius 7.

![Sphere Diagram]

17. The composite solid is a cone with height 6 and base radius 8 and a hemisphere with radius 8.

18. The cross section is a rectangle.

19. The cross section is a square.

20. The cross section is a triangle.

21. The area of the base is \( A = lw = 1.5 \cdot 2.1 = 3.15 \).

\[ V = Bh \]
\[ = 3.15 \cdot 3.6 \]
\[ = 11.34 \]

The volume of the rectangular prism is 11.34 cubic meters.

22. The area of the base is \( A = \pi r^2 = \pi (2)^2 = 4\pi \).

\[ V = Bh \]
\[ = 4\pi \cdot 8 \]
\[ = 32\pi \approx 100.53 \]

The volume of the cylinder is about 100.53 cubic millimeters.

23. Area of pentagonal base:

The central angle of the regular pentagon is \( \frac{360^\circ}{5} = 72^\circ \).

The apothem bisects the central angle, so each bisected angle measures 36°.

\[ \tan 36^\circ = \frac{1}{a} \]
\[ a = \frac{1}{\tan 36^\circ} \approx 1.38 \]

The apothem is about 1.38.

\[ A = \frac{1}{2}a \cdot ns \]
\[ = \frac{1}{2} (1.38)(5)(2) \]
\[ = 6.9 \]

\[ V = Bh \]
\[ = (6.9)(4) \]
\[ = 27.6 \]

The volume of the regular pentagonal prism is about 27.6 cubic yards.

24. \( V = \frac{1}{3}Bh \)

\[ = \frac{1}{3} \cdot 9^2 \cdot 7 \]
\[ = \frac{1}{3} \cdot 81 \cdot 7 = 189 \]

The volume of the pyramid is 189 cubic feet.

25. \( V = \frac{1}{3}Bh \)

\[ = \frac{1}{3} \cdot \left( \frac{1}{2} \cdot 8 \cdot 15 \right) \cdot 20 \]
\[ = 400 \]

The volume of the pyramid is 400 cubic yards.

26. \( V = \frac{1}{3}Bh \)

\[ = \frac{1}{3} \cdot 18 \cdot 10 \cdot 5 \]
\[ = 300 \]

The volume of the pyramid is 300 cubic meters.

27. \( V = \frac{1}{3}Bh \)

\[ 60 = \frac{1}{3} \cdot s^2 \cdot 15 \]
\[ 180 = s^2 \]
\[ 15 \]
\[ s = \sqrt{12} \approx 3.46 \]

The side of the base of the square pyramid is about 3.46 inches.
Chapter 11

28. \( V = \frac{1}{3} Bh \)
    
    \[
    1024 = \frac{1}{3} \cdot 16^2 \cdot h
    \]
    
    \[
    \frac{3072}{256} = s^2
    \]
    
    \[
    s = 12
    \]

The side of the height of the pyramid is 12 inches.

29. \( S = \pi r^2 + \pi r \ell \)
    
    \[
    = \pi \cdot 9^2 + \pi \cdot 9 \cdot 15
    \]
    
    \[
    = 216\pi
    \]
    
    \[
    \approx 678.58
    \]

\( V = \frac{1}{3} Bh \)

\[
= \frac{1}{3} \cdot 9^2 \cdot \pi \cdot 12
\]

\[
= \frac{1}{3} \cdot 972\pi
\]

\[
= 324\pi
\]

\[
\approx 1017.88
\]

The surface area is about 678.58 square centimeters and the volume is about 1017.88 cubic centimeters.

30. \( S = \pi r^2 + \pi r \ell \)
    
    \[
    = \pi \cdot 16^2 + \pi \cdot 16 \cdot 34
    \]
    
    \[
    = 256\pi + 544\pi
    \]
    
    \[
    = 800\pi
    \]
    
    \[
    \approx 2513.27
    \]

\( V = \frac{1}{3} Bh \)

\[
= \frac{1}{3} \cdot 16^2\pi \cdot 30
\]

\[
= 10 \cdot 256\pi
\]

\[
= 2560\pi
\]

\[
\approx 8042.48
\]

The surface area is about 2513.27 square centimeters and the volume is about 8042.48 cubic centimeters.

31. \( S = \pi r^2 + \pi r \ell \)
    
    \[
    = \pi \cdot 7^2 + \pi \cdot 7 \cdot 13
    \]
    
    \[
    = 49\pi + 91\pi
    \]
    
    \[
    = 140\pi
    \]
    
    \[
    \approx 439.82
    \]

Find the height using the Pythagorean Theorem.

\[
c^2 = a^2 + b^2
\]

\[
13^2 = 7^2 + h^2
\]

\[
169 = 49 + h^2
\]

\[
120 = h^2
\]

\[
h = \sqrt{120} = 2\sqrt{30}
\]

\( V = \frac{1}{3} Bh \)

\[
= \frac{1}{3} \cdot 7\pi \cdot 2\sqrt{30}
\]

\[
= \frac{1}{3} \cdot 49\pi \cdot 2\sqrt{30}
\]

\[
= 562.10
\]

The surface area is about 439.82 square meters and the volume is about 562.10 cubic meters.

32. The radius is \( 16 \div 2 = 8 \) centimeters.

\( V = \frac{1}{3} Bh \)

\[
320\pi = \frac{1}{3} \cdot (\pi \cdot 8^2) \cdot h
\]

\[
960\pi = 64\pi \cdot h
\]

\[
960\pi
\]

\[
64\pi = h
\]

\[
15 = h
\]

The height of the cone is 15 centimeters.

33. \( S = 4\pi r^2 \)
    
    \[
    = 4\pi(7)^2
    \]
    
    \[
    = 4\pi \cdot 49
    \]
    
    \[
    = 196\pi
    \]
    
    \[
    \approx 615.75
    \]

\( V = \frac{4}{3}\pi r^3 \)

\[
= \frac{4}{3}\pi(7)^3
\]

\[
= \frac{1372}{3}\pi
\]

\[
\approx 1436.76
\]

The surface area is about 615.75 square inches and the volume is about 1436.76 cubic inches.
34. The radius is 17 \div 2 = 8.5 \text{ feet.}

\begin{align*}
S &= 4\pi r^2 \\
&= 4\pi (8.5)^2 \\
&= 4\pi \cdot 72.25 \\
&= 289\pi \\
&\approx 907.92 \\
V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi (8.5)^3 \\
&= \frac{4}{3}\pi \cdot 595.3 \\
&= \frac{2456.5}{3} \\
&\approx 818.85
\end{align*}

The surface area is about 907.92 square feet and the volume is about 818.85 cubic feet.

35. \(C = 2\pi r\)

\[30\pi = 2\pi r\]

\[\frac{30\pi}{2\pi} = r\]

\[15 = r\]

So, the radius is 15 feet.

\begin{align*}
S &= 4\pi r^2 \\
&= 4\pi (15)^2 \\
&= 4\pi \cdot 225 \\
&= 900\pi \\
&\approx 2827.43 \\
V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi (15)^3 \\
&= \frac{4}{3}\pi \cdot 3375 \\
&= \frac{13500}{3} \\
&\approx 4500\pi \\
&\approx 14,137.17
\end{align*}

The surface area is about 2827.43 square feet and the volume is about 14,137.17 cubic feet.

36. The radius is 4880 \div 2 = 2440 \text{ kilometers.}

\begin{align*}
S &= 4\pi r^2 \\
&= 4\pi (2440)^2 \\
&= 4\pi \cdot 5,953,600 \\
&= 23,814,400\pi \\
&\approx 74,815,144.09 \\
V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi (2440)^3 \\
&= \frac{4}{3}\pi \cdot 12,592,000 \\
&\approx 6.085 \times 10^{10}
\end{align*}

The surface area is about 74,815,144.09 square kilometers and the volume is about 6.085 \times 10^{10} cubic kilometers.

37. Volume of cube: \(6^3 = 216\)

Volume of hemisphere: \(\left(\frac{1}{2} \cdot \frac{4}{3}\pi r^3\right) = \left(\frac{2}{3}\pi \cdot 3^3\right) = 18\pi\)

Volume of cube + Volume of hemisphere

= 216 + 18\pi

\approx 272.55

The volume of the composite solid is about 272.55 cubic meters.

Chapter 11 Test (p. 661)

1. Area of hexagon:

The central angle is \(\frac{360\degree}{6} = 60\degree\). Since the triangle formed by the radii of the polygon is isosceles, each base angle is 60\degree. The base (side of the polygon) is divided into two equal parts, 4. The apothem is the shorter leg multiplied by \(\sqrt{3}\); So, \(a = 4\sqrt{3}\).

\[A = \frac{1}{2} \cdot a \cdot ns\]

\[= \frac{1}{2} \cdot 4\sqrt{3} \cdot 6 \cdot 8\]

\[= 96\sqrt{3}\]

\[V = Bh\]

\[= 96\sqrt{3} \cdot 15.5\]

\[\approx 2577.29\]

The volume of the hexagonal prism is about 2577.29 cubic meters.

2. \(V = \frac{4}{3}\pi r^3\)

\[= \frac{4}{3}\pi (1.6)^3\]

\[\approx 17.16\]

The volume of the sphere is about 17.16 cubic feet.

3. The volume of each cone is \(\frac{1}{3} \cdot \frac{1}{2} \cdot \pi \cdot 4^2 \cdot 3 = 50.27\) cubic meters.

The volume of the center cylinder is \(\pi \cdot 4^2 \cdot 6 = 301.59\) cubic meters.

Volume of top cone + Volume of cylinder + Volume of bottom cone

\[= 50.27 + 301.59 + 50.27\]

\[= 402.13\]

The volume of the composite solid is about 402.13 cubic meters.

4. The volume of the pyramid is \(\frac{1}{3} \cdot \pi \cdot 2.5^2 \cdot 4 \approx 13.33\) cubic feet.

The volume of the prism is \(5 \cdot 2 \cdot 8 = 80\) cubic feet.

Volume of pyramid + Volume of prism = 13.33 + 80 = 93.33

The volume of the composite solid is about 93.33 cubic feet.
5. Arc length of $\overline{ED} = \frac{m\overline{ED}}{360^\circ} \cdot C$

   $64 = \frac{210^\circ}{360^\circ} \cdot C$

   $64 \cdot 12 = C$

   $7 = C$

   $109.71 \approx C$

   The circumference of $\odot F$ is about 109.71 inches.

6. Arc length of $\overline{GH} = \frac{m\overline{GH}}{360^\circ} \cdot C$

   $35 = \frac{m\overline{GH}}{360^\circ} \cdot 54\pi$

   $\frac{35 \cdot 360^\circ}{54\pi} = m\overline{GH}$

   $74.27^\circ \approx m\overline{GH}$

7. Area of sector $QTR = \frac{m\overline{QTR}}{360^\circ} \cdot \text{Area of } \odot S$

   $= \frac{360^\circ - 105^\circ}{360^\circ} \cdot \pi \cdot 8^2$

   $\approx 142.42$

   The area of the shaded sector is about 142.42 square inches.

8. The composite solid is a cylinder with height 6 and base radius 3, and a hemisphere with radius 3.

9. Find the slant height.

   $\ell^2 = h^2 + r^2$

   $\ell^2 = 12^2 + 5^2$

   $\ell = \sqrt{169}$

   $\ell = 13$

   $S = \pi r^2 + \pi \ell$

   $= \pi \cdot 5^2 + \pi \cdot 5 \cdot 13$

   $= 25\pi + 65\pi$

   $= 90\pi \approx 282.74$

   The surface area of the cone is about 282.74 square feet.

10. a. $V = \frac{1}{3} \pi r^2 h$

    $= \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 10$

    $= 120\pi \approx 376.99$

    The volume of the funnel is about 376.99 cubic centimeters.

    b. $\frac{376.99 \text{ mL}}{1} \cdot \frac{1 \text{ sec}}{45 \text{ mL}} = \frac{376.99 \text{ sec}}{45} \approx 8.4 \text{ sec}$

    It will take about 8.4 seconds to empty the funnel.

    c. $V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \cdot \pi \cdot 10^2 \cdot 6$

    $= 200 \cdot \pi \approx 628.32 \text{ cm}^3$, or mL

    $628.32 \text{ mL} \cdot \frac{1 \text{ sec}}{45 \text{ mL}} = \frac{628.32 \text{ sec}}{45} = 13.96 \text{ sec}$

    It will take about 13.96 seconds to empty the funnel.

    d. Sample answer: Changing the radius has a greater effect than changing the height.

11. Use the formula for the volume of a cylinder.

    $V = \pi r^2 h$

    $500 = \pi (3.75)^2 h$

    $\frac{500}{14.06\pi} = h$

    $\approx 11.32 \approx h$

    The height of the bottle is about 11.32 centimeters.

12. The central angle of a dodecagon is $\frac{360^\circ}{12} = 30^\circ$. The base angles of the isosceles triangles formed by the radii of the polygon and the central angle are 75° each. Find the apothem.

    $\tan 75^\circ = \frac{a}{4.5}$

    $a = 4.5 \cdot \tan 75^\circ \approx 16.79$

    $A = \frac{1}{2} a \cdot ns$

    $= \frac{1}{2} \cdot 16.79 \cdot 12 \cdot 9$

    $= 906.66$

    The area of the regular dodecagon is about 906.66 square inches.

13. Fan shown (Area of the sector) $\frac{120 \cdot \pi \cdot 9^2}{360} \approx 84.82 \text{ cm}^2$

    Second fan (Area of the sector) $\frac{150 \cdot \pi \cdot 6^2}{360} \approx 47.12 \text{ cm}^2$

    Since the area of the fan shown is greater than the area of the second fan, the fan shown will do a better job of cooling you.
3. a. Volume of cylinder: \( \pi r^2 h = \pi \cdot 4.25^2 \cdot 80 = 144\pi \)

Volume of cone tip: \( \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi \cdot 3.25^2 \cdot 10 = \frac{845}{24} \pi \)

Volume of cylinder + Volume of cone tip
\[
= 144\pi + \frac{845}{24} \pi
\]
\[
= 4650.2
\]
The volume of the crayon is about 4650.2 cubic millimeters.

b. The volume of the crayon box is
\[
94 \cdot 28 \cdot 71 = 186,872 \text{ cubic millimeters.}
\]
The amount of space not taken up by the 24 crayons is about \( 186,872 - 24 \cdot 4650.2 = 75,267.2 \) cubic millimeters.

4. A; \( x + \frac{1}{2} y = -1 \)
   \[
y = -2x - 2
\]
The slope of the line parallel to \( y = -2x - 2 \) is \( m = -2 \).
   \[
y = -2x + b
\]
6 = -2 • 2 + b
9 = b
The equation of the line parallel to \( y = -2x - 2 \) is
\[
y = -2x + 9
\]

5. A; \( V = \frac{1}{3} Bh \)
   \[
   = \frac{1}{3} \cdot 34.5 \cdot 34.5 \cdot 55.5
   \]
   \[
   = 22,019.63
   \]
The volume of the pyramidion is about 22,019.63 cubic feet.

6. \( r^2 = (x - h)^2 + (y - k)^2 \)
   \( r^2 = (0 - 0)^2 + (2 - 0)^2 \)
   \( r^2 = 4 \)
   \( r = 2 \)
   \( r^2 = (x - h)^2 + (y - k)^2 \)
   \( 2^2 = (1 - 0)^2 + (\sqrt{3} - 0)^2 \)
   \( 4 = 1 + 3 \)
   \( 4 = 4 \checkmark \)
Since the radius is 2 and the distance between the center and the point \((1, \sqrt{3})\) is equal to the radius, then the point \((1, \sqrt{3})\) lies on the circle.

7. yes; Sample answer: The bottom part of the house has parallel rectangular bases at the bottom and top, and the top part of the house has parallel triangular bases on two of the sides.

8. Volume of pyramid = Volume of the cone
   \[
   \frac{1}{3} s^2 h = \frac{1}{3} \pi r^2 h
   \]
   \[
   s = r\sqrt{\pi}
   \]
Both solids will have the same volume if the square base has sides of length \( r\sqrt{\pi} \).

9. The area of the region is about \( \pi s^2 = 25\pi \approx 78.54 \) square miles.
   Population density:
   \[
   \frac{\text{Number of people}}{\text{Area of land}} = \frac{19,400}{78.54} \approx 247
   \]
The population density is about 247 people per square mile.